

INFORMED SEARCH 2

David Kauchak
CS51A – Spring 2019

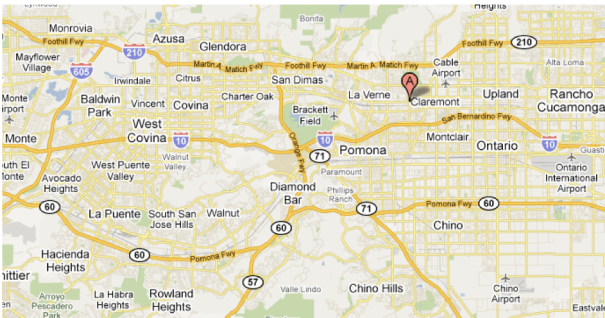
Admin

Assignment 9

Assignment 10

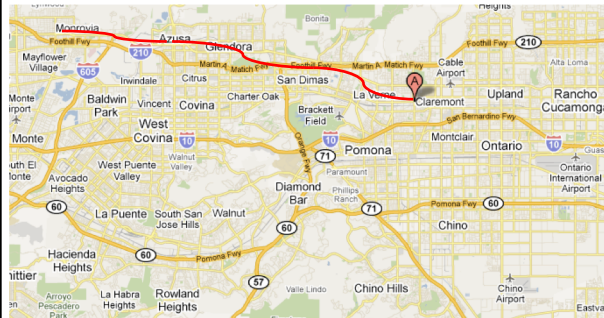
from: Claremont to: Rowland Heights

What would the search algorithms do?

A map of Southern California showing a route from Claremont to Rowland Heights. The route is highlighted in red and follows major highways: 210, 60, 78, 60, 57, and 60. The route starts in Claremont, goes east to Upland, then south to Diamond Bar, then south to Chino, then south to Rowland Heights.

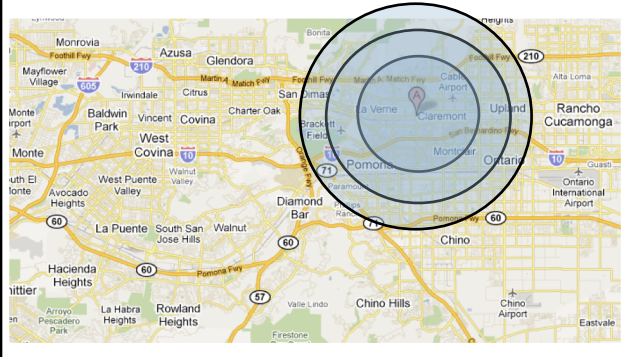
from: Claremont to: Rowland Heights

DFS

A map of Southern California showing a route from Claremont to Rowland Heights. The route is highlighted in red and follows major highways: 210, 60, 78, 60, 57, and 60. The route starts in Claremont, goes east to Upland, then south to Diamond Bar, then south to Chino, then south to Rowland Heights.

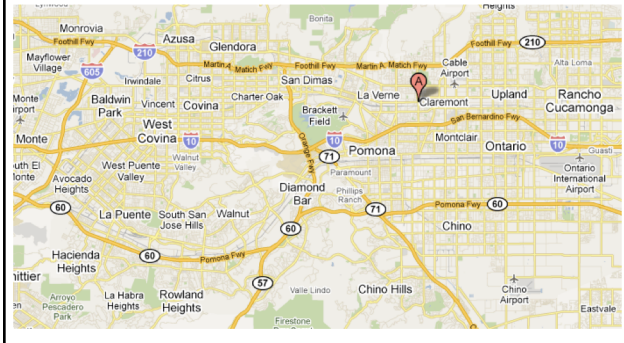
from: Claremont to: Rowland Heights

BFS



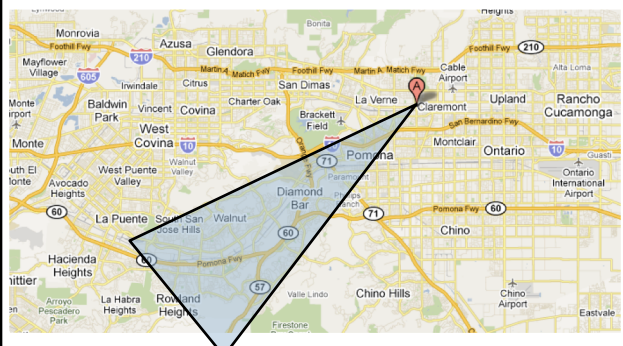
from: Claremont to: Rowland Heights

Ideas?



from: Claremont to: Rowland Heights

We'd like to bias search towards the actual solution



Informed search

Order to visit based on some knowledge of the world that estimates how "good" a state is

- $h(n)$ is called an evaluation function

Best-first search

- rank to visit based on $h(n)$
- take the most desirable state in to visit first
- different approaches depending on how we define $h(n)$

Heuristic

Merriam-Webster's Online Dictionary

Heuristic (pron. \hyu- 'ris-tikl): adj. [from Greek *heuriskein* to discover.] involving or serving as an aid to learning, discovery, or problem-solving by experimental and especially trial-and-error methods

The Free On-line Dictionary of Computing (2/19/13)

heuristic 1. Of or relating to a usually speculative formulation serving as a guide in the investigation or solution of a problem: "The historian discovers the past by the judicious use of such a heuristic device as the 'ideal type'" (Karl J. Weintraub).

Heuristic function: $h(n)$

An estimate of how close the node is to a goal

Uses domain-specific knowledge!

Examples

- ▣ Map path finding?
- ▣ 8-puzzle?
- ▣ Missionaries and cannibals?

Heuristic function: $h(n)$

An estimate of how close the node is to a goal

Uses domain-specific knowledge!

Examples

- ▣ Map path finding?
 - straight-line distance from the node to the goal ("as the crow flies")
- ▣ 8-puzzle?
 - how many tiles are out of place
 - sum of the "distances" of the out of place tiles
- ▣ Missionaries and cannibals?
 - number of people on the starting bank

Two heuristics

2	8	3
1	6	4
	7	5

1	2	3
8	6	4
	7	5

6	2	3
8		4
7	1	5

Which state is better?

1	2	3
8		4
7	6	5
GOAL		

Two heuristics

2	8	3
1	6	4
	7	5

1	2	3
8		4
7	6	5

Goal

How many tiles are out of place?

Two heuristics

2	8	3
1	6	4
	7	5

1	2	3
8		4
7	6	5

Goal

5

Two heuristics

2	8	3
1	6	4
	7	5

1	2	3
8		4
7	6	5

Goal

What is the "distance" of the tiles that are out of place?

Two heuristics

2	8	3
1	6	4
	7	5

1	2	3
8		4
7	6	5

Goal

6

Two heuristics

	Tiles out of place	Sum of distances for out of place tiles																			
<table border="1"> <tr><td>2</td><td>8</td><td>3</td></tr> <tr><td>1</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	2	8	3	1	6	4		7	5	5	6	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
2	8	3																			
1	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	1	2	3	8	6	4		7	5			<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
1	2	3																			
8	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>6</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>1</td><td>5</td></tr> </table>	6	2	3	8		4	7	1	5												
6	2	3																			
8		4																			
7	1	5																			

?

Two heuristics

	Tiles out of place	Sum of distances for out of place tiles																			
<table border="1"> <tr><td>2</td><td>8</td><td>3</td></tr> <tr><td>1</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	2	8	3	1	6	4		7	5	5	6	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
2	8	3																			
1	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	1	2	3	8	6	4		7	5	2	2	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
1	2	3																			
8	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>6</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>1</td><td>5</td></tr> </table>	6	2	3	8		4	7	1	5	2	6										
6	2	3																			
8		4																			
7	1	5																			

Two heuristics

	Tiles out of place	Sum of distances for out of place tiles																			
<table border="1"> <tr><td>2</td><td>8</td><td>3</td></tr> <tr><td>1</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	2	8	3	1	6	4		7	5	5	6	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
2	8	3																			
1	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	1	2	3	8	6	4		7	5	2	2	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
1	2	3																			
8	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>6</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>1</td><td>5</td></tr> </table>	6	2	3	8		4	7	1	5	2	6										
6	2	3																			
8		4																			
7	1	5																			

Which heuristic is better (if either)?

Two heuristics

	Tiles out of place	Sum of distances for out of place tiles																			
<table border="1"> <tr><td>2</td><td>8</td><td>3</td></tr> <tr><td>1</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	2	8	3	1	6	4		7	5	5	6	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
2	8	3																			
1	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td>6</td><td>4</td></tr> <tr><td> </td><td>7</td><td>5</td></tr> </table>	1	2	3	8	6	4		7	5	2	2	<table border="1"> <tr><td>1</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>6</td><td>5</td></tr> </table> GOAL	1	2	3	8		4	7	6	5
1	2	3																			
8	6	4																			
	7	5																			
1	2	3																			
8		4																			
7	6	5																			
<table border="1"> <tr><td>6</td><td>2</td><td>3</td></tr> <tr><td>8</td><td> </td><td>4</td></tr> <tr><td>7</td><td>1</td><td>5</td></tr> </table>	6	2	3	8		4	7	1	5	2	6										
6	2	3																			
8		4																			
7	1	5																			

More closely approximates "real" number of steps remaining?

2	8	3
1	6	4
7		5

Next states?

2	8	3
1	6	4
7		5

2	8	3
1	6	4
7		5

2	8	3
1		4
7	6	5

2	8	3
1	6	4
7	5	

Which would you do?

2	8	3
1	6	4
7		5

2	8	3
1	6	4
7		5

2	8	3
1		4
7	6	5

2	8	3
1	6	4
7	5	

Which would DFS choose

Completely depends on how next states are generated.
Not an "intelligent" decision!

1	2	3
8		4
7	6	5

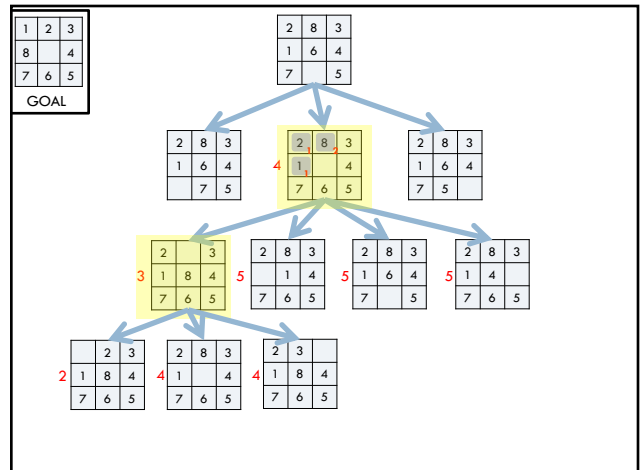
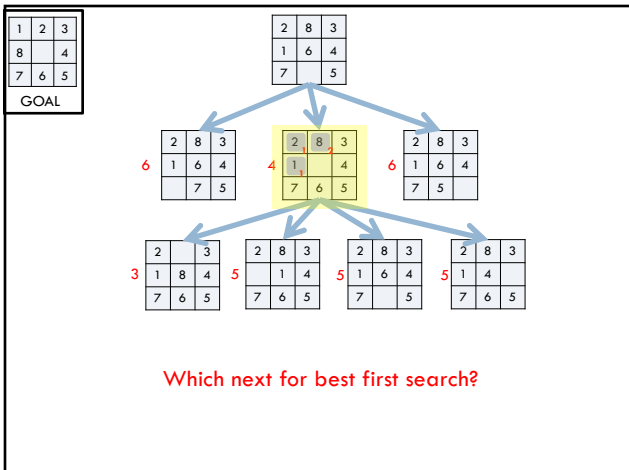
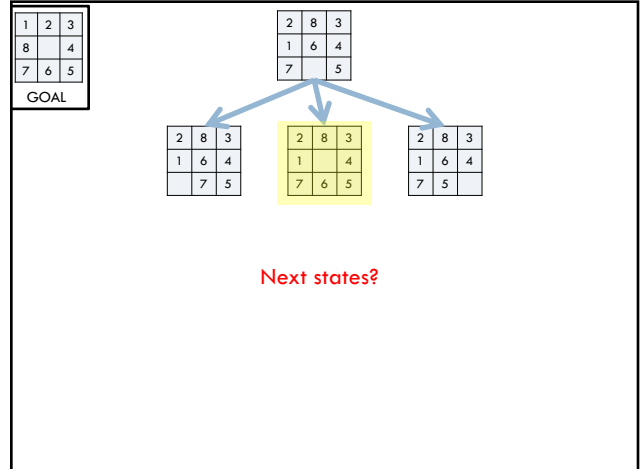
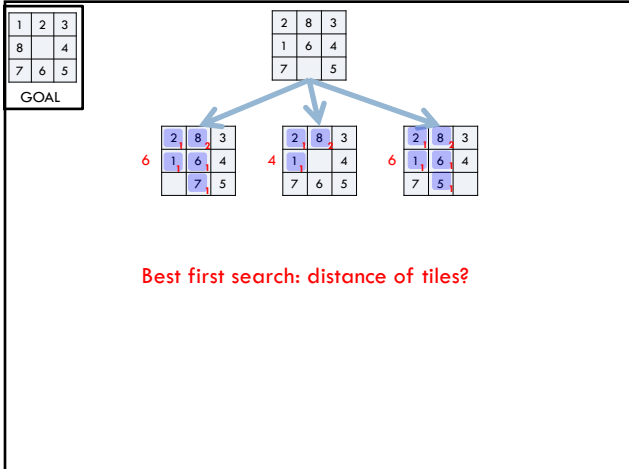
GOAL

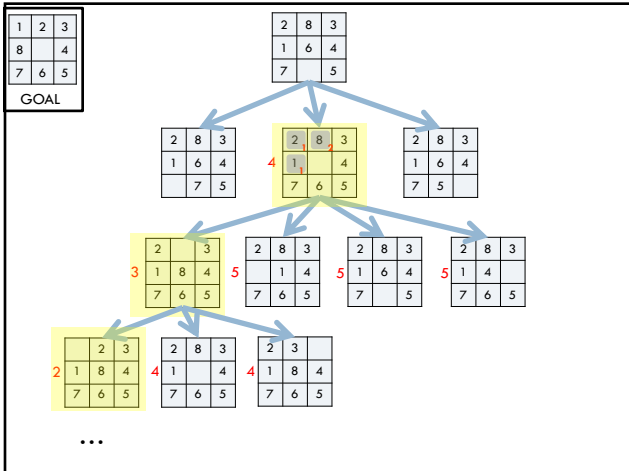
2	8	3
1	6	4
7		5

2	8	3
1		4
7	6	5

2	8	3
1	6	4
7	5	

Best first search: out of place tiles?





Informed search algorithms

Best first search is called an “informed” search algorithm

Why wouldn't we always use an informed algorithm?

- Coming up with good heuristics can be hard for some problems
- There is computational overhead (both in calculating the heuristic and in keeping track of the next “best” state)

Informed search algorithms

Any other problems/concerns about best first search?

Informed search algorithms

Any other problems/concerns about best first search?

- Only as good as the heuristic function

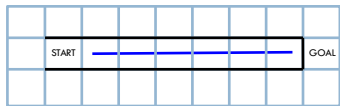
Best first search using distance as the crow flies as heuristic

What would the search do?

Informed search algorithms

Any other problems/concerns about best first search?

- Only as good as the heuristic function



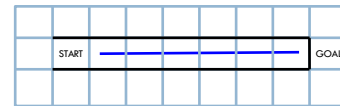
Best first search using distance as the crow flies as heuristic

What is the problem?

Informed search algorithms

Any other problems/concerns about best first search?

- Only as good as the heuristic function



Best first search using distance as the crow flies as heuristic

Doesn't take into account how far it has come.
Best first search is a "greedy" algorithm

Informed search algorithms

Best first search is called an "informed" search algorithm

There are many other informed search algorithms:

- A* search (and variants)
- Theta*
- Beam search

Sudoku

	4	3			6	7	
5			4	2			8
8				6			1
2							5
	5						4
		6				7	
			5	1			
				8			

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

7	2	8	9	3	6	5	1	4
9	4	3	1	5	8	6	7	2
5	6	1	4	7	2	9	3	8
8	3	4	7	6	5	2	9	1
2	1	7	8	4	9	3	6	5
6	5	9	2	1	3	8	4	7
1	8	6	3	2	4	7	5	9
3	7	2	5	9	1	4	8	6
4	9	5	6	8	7	1	2	3

- Fill in the grid with the numbers 1-9
- each row has 1-9 (without repetition)
 - each column has 1-9 (without repetition)
 - each quadrant has 1-9 (without repetition)

Sudoku

	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6				7		
			5		1			
				8				

How can we pose this as a search problem?

State

Start state

Goal state

State space/transitions

- Fill in the grid with the numbers 1-9
- each row has 1-9 (without repetition)
 - each column has 1-9 (without repetition)
 - each quadrant has 1-9 (without repetition)

Sudoku

	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6				7		
			5		1			
				8				

How can we pose this as a search problem?

State: 9 x 9 grid with 1-9 or empty

Start state:

Goal state:

State space/transitions

- Fill in the grid with the numbers 1-9
- each row has 1-9 (without repetition)
 - each column has 1-9 (without repetition)
 - each quadrant has 1-9 (without repetition)

Sudoku

	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6				7		
			5		1			
				8				

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

- Fill in the grid with the numbers 1-9
- each row has 1-9 (without repetition)
 - each column has 1-9 (without repetition)
 - each quadrant has 1-9 (without repetition)

Sudoku

	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
	6				7			
		5	1					
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

How many next states?
What are they?

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
	6				7			
		5	1					
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

1, 6, 7, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1								
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
	6				7			
		5	1					
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

① 6, 7, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1								
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
	6				7			
		5	1					
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

How many next states?
What are they?

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1								
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6			7			
			5	1				
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

2, 6, 7, 8, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2							
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6			7			
			5	1				
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

2, 6, 7, 8, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2							
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6			7			
			5	1				
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

What are the next states?

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2							
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5					4		
		6			7			
			5	1				
			8					

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

7, 8, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2	7						
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
		6				7		
			5		1			
				8				

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

7, 8, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2	7						
4	3					6	7	
5			4	2				8
8				6				1
2								5
	5						4	
		6				7		
			5		1			
				8				

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2	7						
9	4	3				6	7	
5	6		4	2				8
8				6				1
2								5
	5						4	
		6				7		
			5		1			
				8				

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2	7						
9	4	3				6	7	
5	6		4	2				8
8				6				1
2								5
	5						4	
		6				7		
			5		1			
				8				

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

Now what?

Try another branch, i.e. go back to a place where we had a decision and try a different one

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Sudoku

1	2	8						
	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
		6				7		
			5	1				
				8				

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

7, 8, 9

Fill in the grid with the numbers 1-9

- each row has 1-9 (without repetition)
- each column has 1-9 (without repetition)
- each quadrant has 1-9 (without repetition)

Best first Sudoku search

DFS and BFS will choose entries (and numbers within those entries) randomly

Is that how people do it?

How do you do it?

Heuristics for best first search?

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

Best first Sudoku search

DFS and BFS will choose entries (and numbers within those entries) randomly

Pick the entry that is **MOST** constrained

People often try and find entries where only one option exists and only fill it in that way (very little search)

Generate next states:

- pick an open entry
- try all possible numbers that meet constraints

Representing the Sudoku board

	4	3				6	7	
5			4	2				8
8				6				1
2								5
	5						4	
		6				7		
			5	1				
				8				

Which is the most constrained (of the ones above)?

- Board is a matrix (list of lists)
- Each entry is either:
 - a number (if we've filled in the space already, either during search or as part of the starting state)
 - a list of numbers that are valid to put in that entry if it hasn't been filled in yet

Representing the Sudoku board

	4	3			6	7	
5		4		2			8
8				6			1
2							5
	5					4	
		6			7		
		5		1			
			8				

[1, 6, 7, 9], [1, 2, 6, 7, 8, 9], [1, 2, 7, 8, 9],
 [1, 9], 4, 3,
 5, [1, 6, 7, 9], [1, 7, 9]

Which is the most constrained (of the ones above)?

- Board is a matrix (list of lists)
- Each entry is either:
 - a number (if we've filled in the space already, either during search or as part of the starting state)
 - a list of numbers that are valid to put in that entry if it hasn't been filled in yet

Representing the Sudoku board

1	4	3			6	7	
5		4		2			8
8				6			1
2							5
	5					4	
		6			7		
		5		1			
			8				

What would the state look like if we add pick 1?

- Board is a matrix (list of lists)
- Each entry is either:
 - a number (if we've filled in the space already, either during search or as part of the starting state)
 - a list of numbers that are valid to put in that entry if it hasn't been filled in yet

Representing the Sudoku board

1	4	3			6	7	
5		4		2			8
8				6			1
2							5
	5					4	
		6			7		
		5		1			
			8				

[6, 7, 9], [2, 6, 7, 8, 9], [2, 7, 8, 9],
 [9], 4, 3,
 5, [6, 7, 9], [7, 9]

Remove 1 from all entries in the quadrant

What other parts of the board need to be updated?

- Board is a matrix (list of lists)
- Each entry is either:
 - a number (if we've filled in the space already, either during search or as part of the starting state)
 - a list of numbers that are valid to put in that entry if it hasn't been filled in yet

Representing the Sudoku board

1	4	3			6	7	
5		4		2			8
8				6			1
2							5
	5					4	
		6			7		
		5		1			
			8				

[6, 7, 9], [2, 6, 7, 8, 9], [2, 7, 8, 9],
 [9], 4, 3,
 5, [6, 7, 9], [7, 9]

Remove 1 from all entries in the quadrant

Remove 1 from all entries in the same column

Remove 1 from all entries in the same row

- Board is a matrix (list of lists)
- Each entry is either:
 - a number (if we've filled in the space already, either during search or as part of the starting state)
 - a list of numbers that are valid to put in that entry if it hasn't been filled in yet