

# GRAMMARS

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CS159 – Spring 2019

some slides adapted from  
Ray Mooney

## Admin

Assignment 3 out: due next Monday

Quiz #1

## Context free grammar

$$S \rightarrow NP VP$$

left hand side (single symbol)      right hand side (one or more symbols)

## Formally...

$G = (NT, T, P, S)$

NT: finite set of nonterminal symbols

T: finite set of terminal symbols, NT and T are disjoint

P: finite set of productions of the form  
 $A \rightarrow \alpha, A \in NT \text{ and } \alpha \in (T \cup NT)^*$

$S \in NT$ : start symbol

## CFG: Example

Many possible CFGs for English, here is an example (fragment):

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$   
 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow \text{boy} \mid \text{girl}$   
 $V \rightarrow \text{sees} \mid \text{likes}$   
 $Adj \rightarrow \text{big} \mid \text{small}$   
 $Adv \rightarrow \text{very}$   
 $DetP \rightarrow a \mid \text{the}$

## Derivations in a CFG

$S \rightarrow NP VP$   
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S

What can we do?

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NP VP

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the boy VP

### Derivations in a CFG

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$       the boy likes NP  
 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow boy \mid girl$   
 $V \rightarrow sees \mid likes$   
 $Adj \rightarrow big \mid small$   
 $Adv \rightarrow very$   
 $DetP \rightarrow a \mid the$

### Derivations in a CFG

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$       the boy likes a girl  
 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow boy \mid girl$   
 $V \rightarrow sees \mid likes$   
 $Adj \rightarrow big \mid small$   
 $Adv \rightarrow very$   
 $DetP \rightarrow a \mid the$

### Derivations in a CFG; Order of Derivation Irrelevant

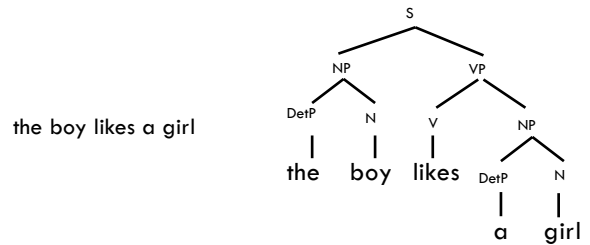
$S \rightarrow NP VP$   
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 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow boy \mid girl$   
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 $Adv \rightarrow very$   
 $DetP \rightarrow a \mid the$

$NP VP$   
 $\swarrow \quad \searrow$   
 $DetP N VP \quad NP V NP$   
 $\Downarrow$   
 the boy likes a girl

### Derivations of CFGs

String rewriting system: we derive a string

Derivation history shows constituent tree:



## Parsing

Parsing is the field of NLP interested in automatically determining the syntactic structure of a sentence

Parsing can be thought of as determining what sentences are "valid" English sentences

As a byproduct, we often can get the structure

## Parsing

Given a CFG and a sentence, determine the possible parse tree(s)

S -> NP VP  
 NP -> N  
 NP -> PRP  
 NP -> N PP  
 VP -> V NP  
 VP -> V NP PP  
 PP -> IN N  
 PRP -> I  
 V -> eat  
 N -> sushi  
 N -> tuna  
 IN -> with

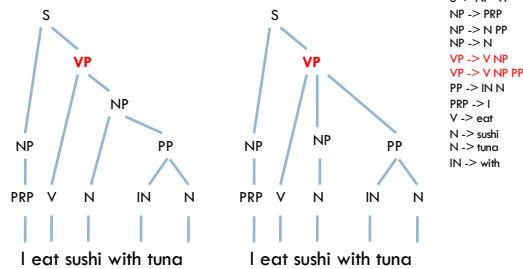
I eat sushi with tuna

What parse trees are possible for this sentence?

How did you do it?

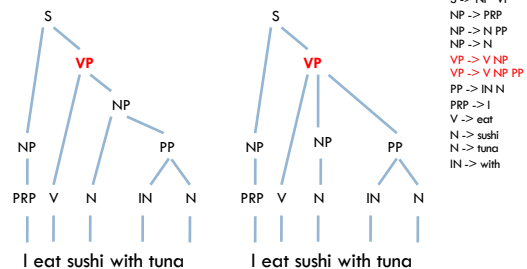
What if the grammar is much larger?

## Parsing



What is the difference between these parses?

## Parsing ambiguity

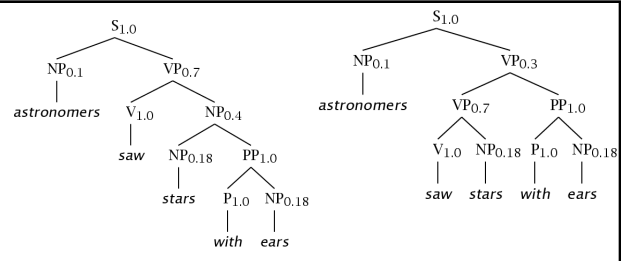


How can we decide between these?

## A Simple PCFG

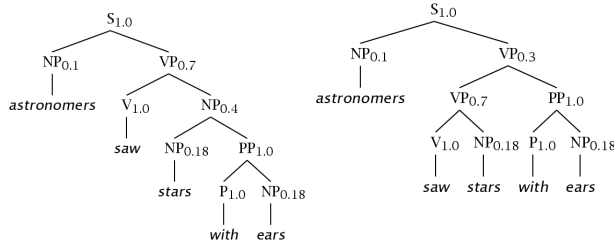
### Probabilities!

S	→	NP VP	1.0	NP	→	NP PP	0.4
VP	→	V NP	0.7	NP	→	<i>astronomers</i>	0.1
VP	→	VP PP	0.3	NP	→	<i>ears</i>	0.18
PP	→	P NP	1.0	NP	→	<i>saw</i>	0.04
P	→	<i>with</i>	1.0	NP	→	<i>stars</i>	0.18
V	→	<i>saw</i>	1.0	NP	→	<i>telescope</i>	0.1



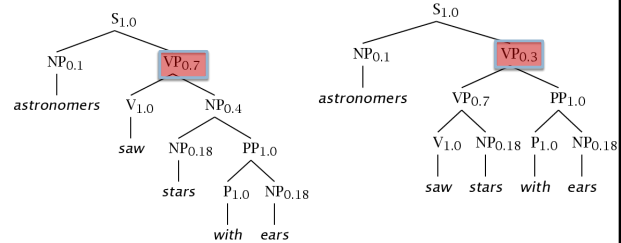
Just like *n*-gram language modeling, PCFGs break the sentence generation process into smaller steps/probabilities

The probability of a parse is the product of the PCFG rules



What are the different interpretations here?

Which do you think is more likely?



$$= 1.0 * 0.1 * 0.7 * 1.0 * 0.4 * 0.18$$

$$= 0.0009072$$

$$= 1.0 * 0.1 * 0.3 * 0.7 * 1.0 * 0.18$$

$$= 0.0006804$$

## Parsing problems

### Pick a model

- e.g. CFG, PCFG, ...

### Train (or learn) a model

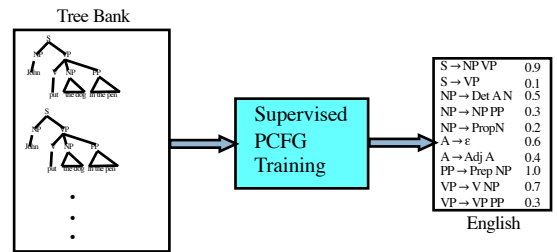
- What CFG/PCFG rules should I use?
- Parameters (e.g. PCFG probabilities)?
- What kind of data do we have?

### Parsing

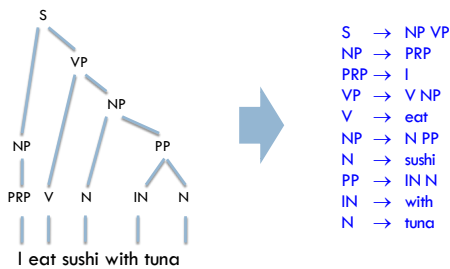
- Determine the parse tree(s) given a sentence

## PCFG: Training

If we have example parsed sentences, how can we learn a set of PCFGs?



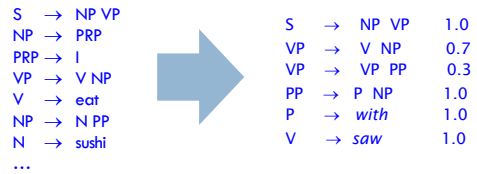
## Extracting the rules



What CFG rules occur in this tree?

## Estimating PCFG Probabilities

We can extract the rules from the trees



How do we go from the extracted CFG rules to PCFG rules?

## Estimating PCFG Probabilities

Extract the rules from the trees

Calculate the probabilities using MLE

$$\alpha \rightarrow \beta \quad \longrightarrow \quad p(\alpha \rightarrow \beta | \alpha)$$

$$P(\alpha \rightarrow \beta | \alpha) = \frac{\text{count}(\alpha \rightarrow \beta)}{\sum_{\gamma} \text{count}(\alpha \rightarrow \gamma)} = \frac{\text{count}(\alpha \rightarrow \beta)}{\text{count}(\alpha)}$$

## Estimating PCFG Probabilities

Occurrences

S	→ NP VP	10
S	→ V NP	3
S	→ VP PP	2
NP	→ N	7
NP	→ N PP	3
NP	→ DT N	6

$P(S \rightarrow V NP) = ?$

$$P(S \rightarrow V NP) = P(S \rightarrow V NP | S) = \frac{\text{count}(S \rightarrow V NP)}{\text{count}(S)} = \frac{3}{15}$$

## Grammar Equivalence

What does it mean for two grammars to be equal?

## Grammar Equivalence

**Weak equivalence:** grammars generate the same set of strings

- Grammar 1: NP → DetP N and DetP → a | the
- Grammar 2: NP → a N | the N

**Strong equivalence:** grammars have the same set of derivation trees

- With CFGs, possible only with useless rules
- Grammar 2: NP → a N | the N
- Grammar 3: NP → a N | the N, DetP → many



## Normal Forms

There are weakly equivalent **normal forms** (Chomsky Normal Form, Greibach Normal Form)

A CFG is in Chomsky Normal Form (CNF) if all productions are of one of two forms:

- $A \rightarrow BC$  with  $A, B, C$  nonterminals
- $A \rightarrow a$ , with  $A$  a nonterminal and  $a$  a terminal

*Every CFG has a weakly equivalent CFG in CNF*

## CNF Grammar

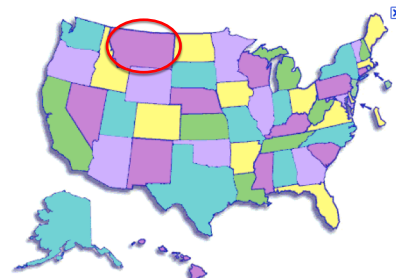
S -> VP	S -> VP
VP -> VB NP	VP -> VB NP
VP -> VB NP PP	VP -> VP2 PP
NP -> DT NN	VP2 -> VB NP
NP -> NN	NP -> DT NN
NP -> NP PP	NP -> NN
PP -> IN NP	NP -> NP PP
DT -> the	PP -> IN NP
IN -> with	DT -> the
VB -> film	IN -> with
VB -> trust	VB -> film
NN -> man	VB -> trust
NN -> film	NN -> man
NN -> trust	NN -> film
	NN -> trust

## Probabilistic Grammar Conversion

Original Grammar                      Chomsky Normal Form

S → NP VP	0.8	S → NP VP	0.8
S → Aux NP VP	0.1	S → X1 VP	0.1
		X1 → Aux NP	1.0
S → VP	0.1	S → book   include   prefer	
		0.01 0.004 0.006	
		S → Verb NP	0.05
		S → VP PP	0.03
NP → Pronoun	0.2	NP → I   he   she   me	
		0.1 0.02 0.02 0.06	
NP → Proper-Noun	0.2	NP → Houston   NWA	
		0.16 .04	
NP → Det Nominal	0.6	NP → Det Nominal	0.6
Nominal → Noun	0.3	Nominal → book   flight   meal   money	
		0.03 0.15 0.06 0.06	
Nominal → Nominal Noun	0.2	Nominal → Nominal Noun	0.2
Nominal → Nominal PP	0.5	Nominal → Nominal PP	0.5
VP → Verb	0.2	VP → book   include   prefer	
		0.1 0.04 0.06	
VP → Verb NP	0.5	VP → Verb NP	0.5
VP → VP PP	0.3	VP → VP PP	0.3
PP → Prep NP	1.0	PP → Prep NP	1.0

## States



What is the capital of this state? Helena (Montana)

## Grammar questions

Can we determine if a sentence is grammatical?

Given a sentence, can we determine the syntactic structure?

Can we determine how likely a sentence is to be grammatical? to be an English sentence?

Can we generate candidate, grammatical sentences?

Next time:  
parsing