

CS41B MACHINE

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CS 52 – Spring 2016

Admin

- Assignment 3
 - due Monday at 11:59pm
- Academic honesty

A few rules to follow for this course to keep you out of trouble:

- If you talk with someone in the class about a problem, you should *not* take notes. If you understand the material you talked about, you should be able to recreate it on your own.
- Similarly, if you talk with someone, you must wait 5 minutes before resuming work on the problem. Stretch. Use the restroom. Go for a quick walk. This will ensure that you really understand the material.
- You may not sit next to (or where you can see the screen of) anyone you are talking with about the assignment.
- The only time you may look at someone else's screen when they are working on an assignment is if they are asking *you* for help with a syntax error. You should not look at someone else's code to help yourself!

If you are ever unsure about what constitutes acceptable collaboration, please ask!

Admin

Midterm next Thursday in-class (2/18)

- Comprehensive ☺
- Closed books, notes, computers, etc.
- **Except**, may bring up to 2 pages of notes
- Practice problems posted
 - Also some practice problems in the Intro SML reading
- Midterm review sessions (will announce on piazza soon)

Midterm topics

SML

recursion

math

Midterm topics

- Basic syntax
- SML built-in types
- Defining function
 - pattern matching
- Function type signatures
- Recursion!
- map
- exceptions
- Defining datatypes
- addition, subtraction, multiplication manually and on list digits
- Numbers in different bases
- Binary number representation (first part of today's lecture)
- NOT CS41B material

Binary number revisited

What number does 1001 represent in binary?

Depends!

Is it a signed number or unsigned?

If signed, what convention are we using?

Twos complement

For a number with n digits high order bit represents -2^{n-1}

unsigned

2 ³	2 ²	2 ¹	2 ⁰
----------------	----------------	----------------	----------------

signed

(twos complement)

-2 ³	2 ²	2 ¹	2 ⁰
-----------------	----------------	----------------	----------------

Twos complement

What number is it?

unsigned

1	0	0	1	
2 ³	2 ²	2 ¹	2 ⁰	9

signed

(twos complement)

1	0	0	1	
-2 ³	2 ²	2 ¹	2 ⁰	-7

Twos complement

What number is it?

unsigned	1	1	1	1	15
	2^3	2^2	2^1	2^0	
signed (twos complement)	1	1	1	1	-1
	-2^3	2^2	2^1	2^0	

Twos complement

What number is it?

unsigned	1	1	0	0	12
	2^3	2^2	2^1	2^0	
signed (twos complement)	1	1	0	0	-4
	-2^3	2^2	2^1	2^0	

Twos complement

How many numbers can we represent with each approach using 4 bits?

16 (2^4) numbers, 0000, 0001, ..., 1111
Doesn't matter the representation!

unsigned	2 ³	2 ²	2 ¹	2 ⁰
signed (twos complement)	-2^3	2 ²	2 ¹	2 ⁰

Twos complement

How many numbers can we represent with each approach using 32 bits?

$2^{32} \approx 4$ billion numbers

unsigned	2 ³	2 ²	2 ¹	2 ⁰
signed (twos complement)	-2^3	2 ²	2 ¹	2 ⁰

Twos complement

What is the range of numbers we can represent for each approach with 4 bits?

unsigned: 0, 1, ... 15

signed: -8, -7, ..., 7

unsigned

2^3	2^2	2^1	2^0
-------	-------	-------	-------

signed

(twos complement)

-2^3	2^2	2^1	2^0
--------	-------	-------	-------

binary representation	unsigned	
0000	0	
0001	1	
0010	?	
0011		
0100		
0101		
0110		
0111		
1000		
1001		
1010		
1011		
1100		
1101		
1110		
1111		

binary representation	unsigned	twos complement
0000	0	?
0001	1	
0010	2	
0011	3	
0100	4	
0101	5	
0110	6	
0111	7	
1000	8	
1001	9	
1010	10	
1011	11	
1100	12	
1101	13	
1110	14	
1111	15	

binary representation	unsigned	twos complement
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	?
1001	9	
1010	10	
1011	11	
1100	12	
1101	13	
1110	14	
1111	15	

binary representation	unsigned	twos complement
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	?
1010	10	
1011	11	
1100	12	
1101	13	
1110	14	
1111	15	

binary representation	unsigned	twos complement
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

binary representation	unsigned	twos complement
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

How can you tell if a number is negative?

binary representation	unsigned	twos complement
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

High order bit!

A two's complement trick

You can also calculate the value of a negative number represented as two's complement as follows:

- Flip all of the bits (0 → 1 and 1 → 0)
- Add 1
- Resulting number is the magnitude of the original negative number

1101 $\xrightarrow{\text{flip the bits}}$ 0010 $\xrightarrow{\text{add 1}}$ 0011 \rightarrow -3

A two's complement trick

You can also calculate the value of a negative number represented as two's complement as follows:

- Flip all of the bits (0 → 1 and 1 → 0)
- Add 1
- Resulting number is the magnitude of the original negative number

1110 $\xrightarrow{\text{flip the bits}}$ 0001 $\xrightarrow{\text{add 1}}$ 0010 \rightarrow -2

Addition with two's complement numbers

$$\begin{array}{r} 0001 \\ + 0101 \\ \hline ? \end{array}$$

Addition with two's complement numbers

$$\begin{array}{r} \\ 0001 \\ + 0101 \\ \hline 0110 \end{array}$$

Addition with twos complement numbers

$$\begin{array}{r} 0110 \\ + 0101 \\ \hline ? \end{array}$$

Addition with twos complement numbers

$$\begin{array}{r} \overset{1}{0}110 \\ + 0101 \\ \hline 1011? \end{array}$$

Addition with twos complement numbers

$$\begin{array}{r} \overset{1}{0}110 \quad 6 \\ + 0101 \quad 5 \\ \hline 1011? \quad 11 \end{array}$$

Overflow! We cannot represent this number (it's too large)

Addition with twos complement numbers

$$\begin{array}{r} 0110 \\ + 1101 \\ \hline ? \end{array}$$

Addition with two's complement numbers

$$\begin{array}{r} \overset{1}{0}110 \\ + \overset{1}{1}101 \\ \hline 0011 \end{array}$$

Addition with two's complement numbers

$$\begin{array}{r} \overset{1}{0}110 \quad 6 \\ + \overset{1}{1}101 \quad -3 \\ \hline 0011 \quad 3 \end{array}$$

ignore the last carry

Subtraction

Ideas?

Subtraction

- Negate the 2nd number (flip the bits and add 1)
- Add them!

Hexadecimal numbers

Hexadecimal = base 16

What will be the digits?

Hexadecimal numbers

Hexadecimal = base 16

Digits

- 0
- 1
- 2
- ...
- 9
- a (10)
- b (11)
- c (12)
- d (13)
- e (14)
- f (15)

What number is 1ad?

Hexadecimal numbers

Hexadecimal = base 16

Digits

- 0
- 1
- 2
- ...
- 9
- a (10)
- b (11)
- c (12)
- d (13)
- e (14)
- f (15)

$1 \ a \ d = 256 + 10 * 16 + 13 = 429$

$16^2 \ 16^1 \ 16^0 = 429$

Hexadecimal numbers

Hexadecimal = base 16

Digits

- 0
- 1
- 2
- ...
- 9
- a (10)
- b (11)
- c (12)
- d (13)
- e (14)
- f (15)

Hexadecimal is common in CS.
Why?

Hexadecimal numbers

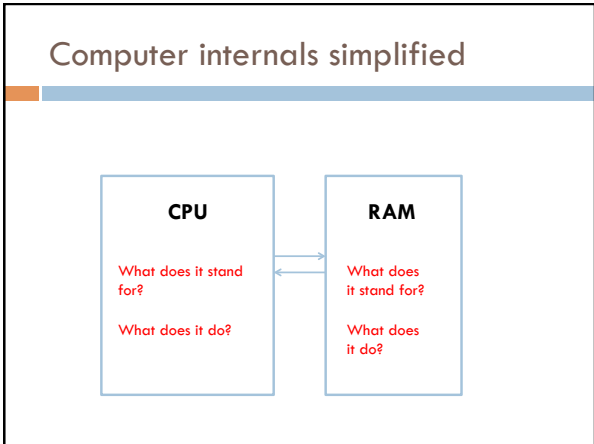
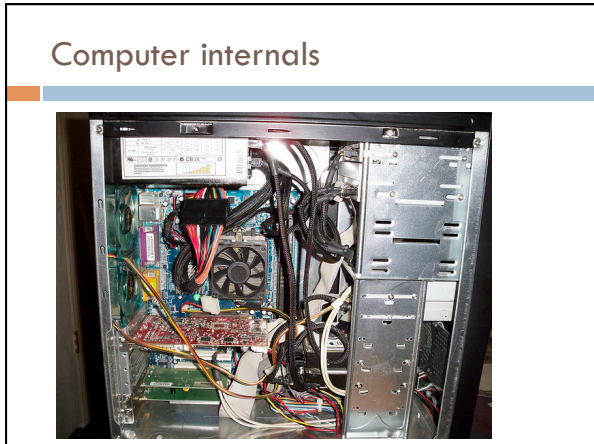
Hexadecimal = base 16

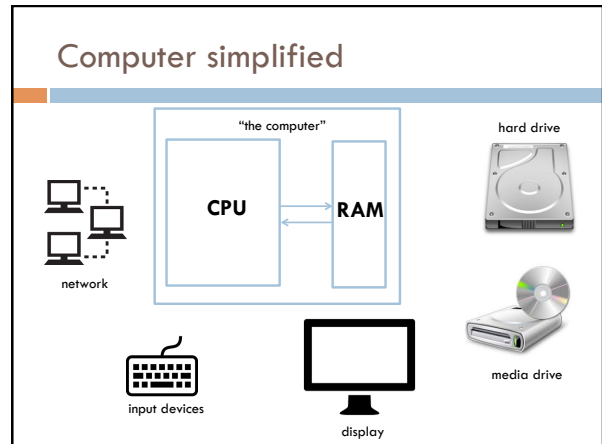
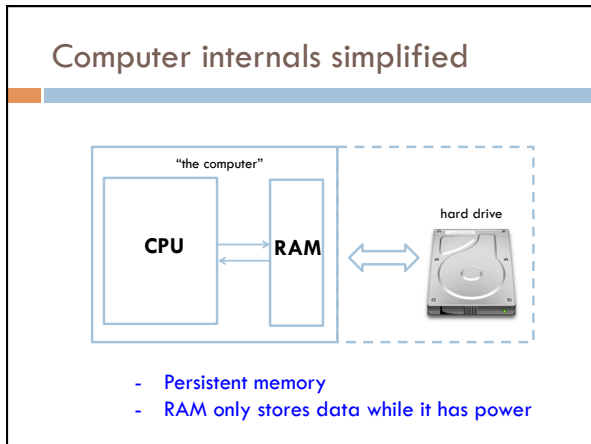
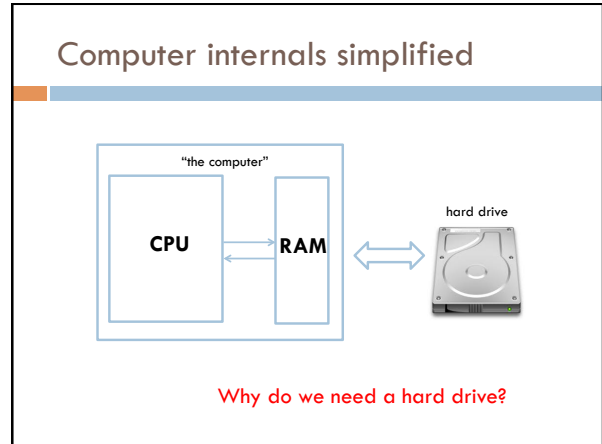
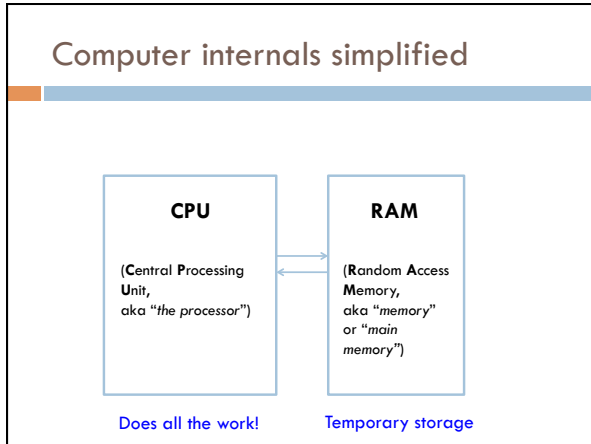
Digits	
0	
1	Hexadecimal is common in CS.
2	4 bits = 1 hexadecimal digit
...	
9	
a (10)	What is the following number in hex?
b (11)	
c (12)	
d (13)	0110 1101 0101 0001
e (14)	
f (15)	

Hexadecimal numbers

Hexadecimal = base 16

Digits	
0	
1	Hexadecimal is common in CS.
2	4 bits = 1 hexadecimal digit
...	
9	
a (10)	What is the following number in hex?
b (11)	
c (12)	
d (13)	0110 1101 0101 0001
e (14)	
f (15)	6 d 5 1







Inside the CPU

CPU

processor





registers

processor: does the work

registers: local, fast memory slots

Why all these levels of memory?

Memory speed


operation	access time	times slower than register access	for comparison ...
register	0.3 ns	1	1 s
RAM	120 ns	400	6 min
Hard disk	1 ms	~million	1 month
google.com	0.4s	~billion	30 years

Memory

RAM

➔

010101111000101000010010 ...



?

What is a byte?

Memory

RAM

➔

{ 01010111 10001010 00010010 ...

byte = 8 bits
byte is abbreviated as B

My laptop has 16GB (gigabytes) of memory. How many bits is that?

Memory sizes

	bits
byte	8
kilobyte (KB)	2^{10} bytes = ~8,000
megabyte (MB)	2^{20} = ~ 8 million
gigabyte (GB)	2^{30} = ~8 billion

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gigabyte (GB)	2^{30} = ~8 billion

~128 billion bits!

Memory

address	
0	01010111
1	10001010
2	00010010
3	01011010
...	...

Memory is byte addressable

Memory

address	
0	01010111
1	10001010
2	00010010
3	01011010
...	...

Memory is organized into “words”, which is the most common functional unit

Memory

address	32-bit words
0	10101011 10001010 00010010 01011010
4	11001011 00001110 01010010 01010110
8	10111011 10010010 00000000 01110100
...	...

Most modern computers use 32-bit (4 byte) or 64-bit (8 byte) words

Memory in the CS41B Machine

address	16-bit words
0	10101011 10001010
2	00010010 01011010
4	11001011 00001110
...	...

We'll use 16-bit words for our model (the CS41B machine)

CS41B machine

CPU

- processor
- registers
- ic: instruction counter (location in memory of the next instruction in memory)
- r0: holds the value 0 (read only)
- r1, r2, r3: general purpose, read/write

CS41B instructions

CPU

- processor
- registers

What types of operations might we want to do (think really basic)?

abbreviation	arguments	action
Register Instructions		
mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

abbreviation	arguments	action
Register Instructions		
mov	RR-	dest = src0
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add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

operation name
(always three characters)

abbreviation	arguments	action
Register Instructions		
mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

operation arguments
R = register (e.g. r0)
S = signed number (byte)

abbreviation	arguments	action
Register Instructions		
mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

operation function
dest = first register
src0 = second register
src1 = third register
arg = number/argument

add r1 r2 r3

What does this do?

abbreviation	arguments	action
--------------	-----------	--------

Register Instructions

mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

add r1 r2 r3

$r1 = r2 + r3$

Add contents of registers r2 and r3 and store the result in r1

abbreviation	arguments	action
--------------	-----------	--------

Register Instructions

mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

adc r2 r1 10

What does this do?

abbreviation	arguments	action
--------------	-----------	--------

Register Instructions

mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

adc r2 r1 10

$r2 = r1 + 10$

Add 10 to the contents of register r1 and store in r2

abbreviation	arguments	action
--------------	-----------	--------

Register Instructions

mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg


```

adc r1 r0 8
neg r2 r1
sub r2 r1 r2

```

What number is in r2?

abbreviation	arguments	action
--------------	-----------	--------

Register Instructions

mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg

```

adc r1 r0 8      r1 = 8
neg r2 r1        r2 = -8, r1 = 8
sub r2 r1 r2     r2 = 16

```

abbreviation	arguments	action
--------------	-----------	--------

Register Instructions

mov	RR-	dest = src0
neg	RR-	dest = -src0
add	RRR	dest = src0 + src1
sub	RRR	dest = src0 - src1
adc	RRS	dest = src0 + arg
sbc	RRS	dest = src0 - arg