Quicksort

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cs201
Spring 2014

```
public static int partition(double[] nums, int start, int end){
  int lessThanIndex = start-1;

  for( int i = start; i < end; i++ ){
    if( nums[i] <= nums[end] ){
      lessThanIndex++;
      swap(nums, lessThanIndex, i);
    }
  }

  swap(nums, lessThanIndex+1, end);

  return lessThanIndex+1;
}
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what does this method do?

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  }
  swap(nums, lessThanIndex+1, end);
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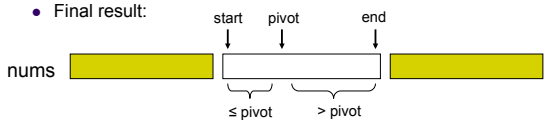
}
```

- nums[end] is called the ***pivot***
- Partitions the elements nums[start…end-1] in to two sets, those ≤ pivot and those > pivot
- Operates in place
- Final result:



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start end

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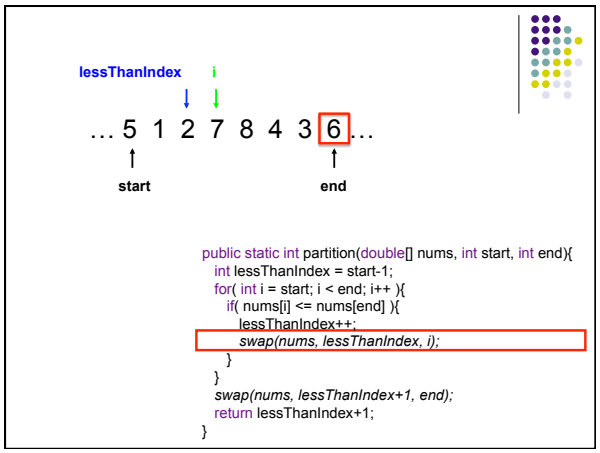


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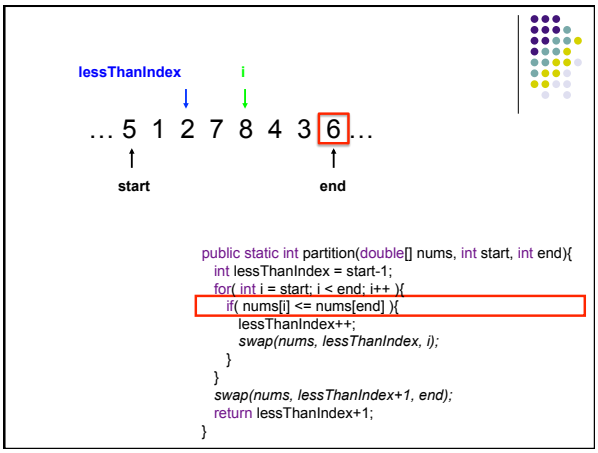


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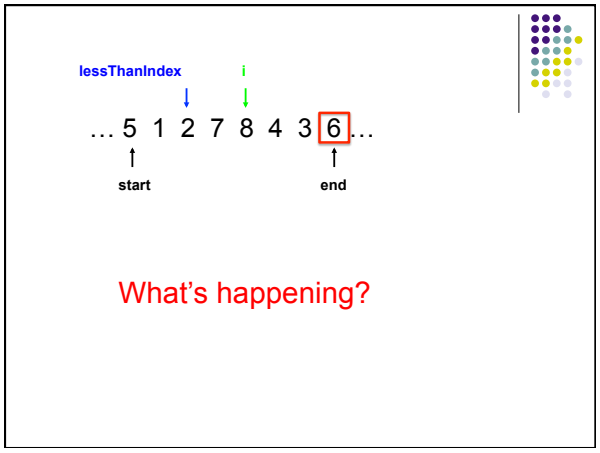

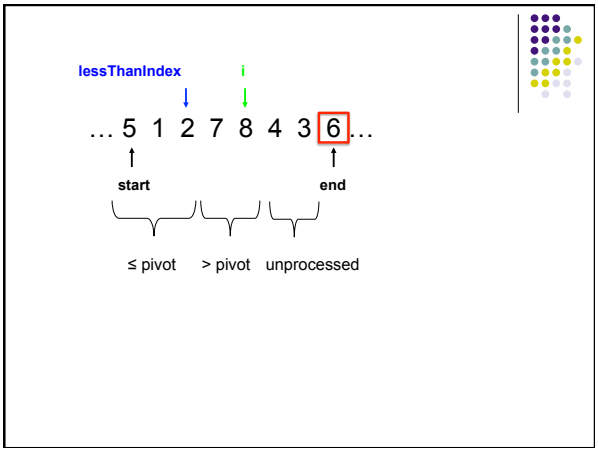

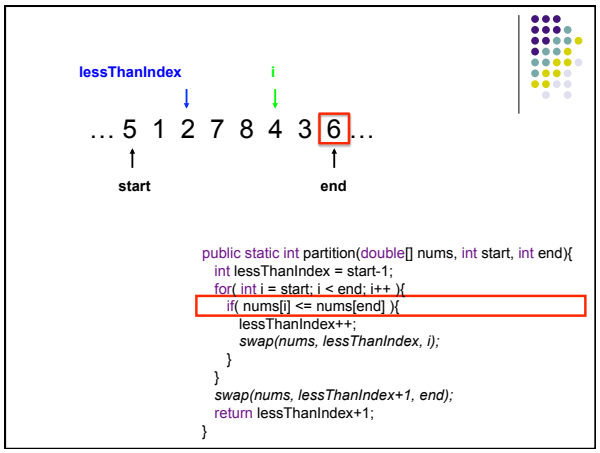


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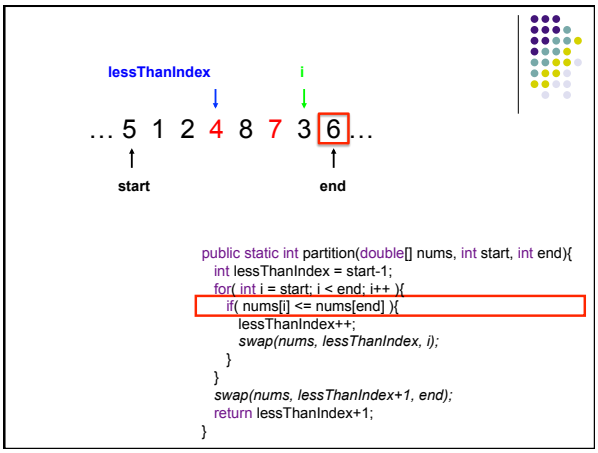


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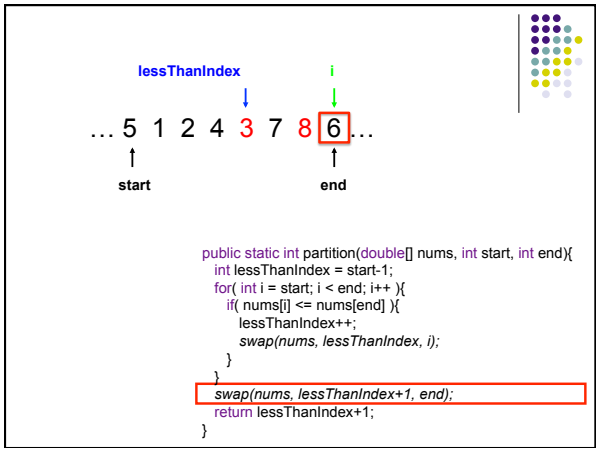


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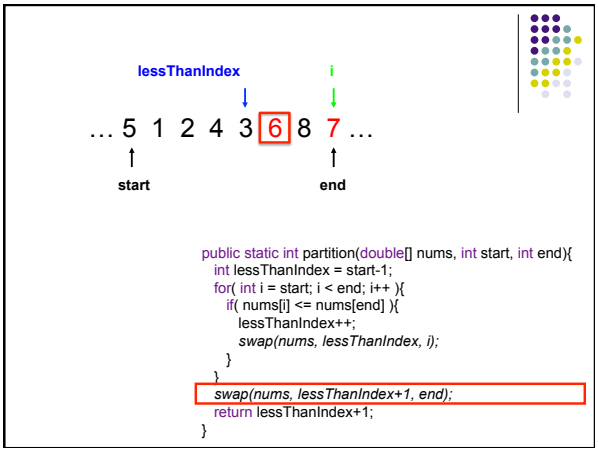


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Partition running time?

O(n)

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Quicksort

How can we use this method to sort nums?

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Quicksort

```
private static void quicksortHelper(double[] nums, int start, int end){
  if( start < end ){
    int partition = partition(nums, start, end);
    quicksortHelper(nums, start, partition-1);
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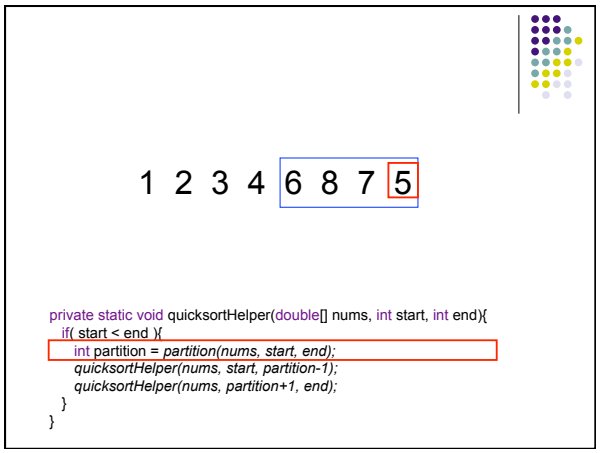
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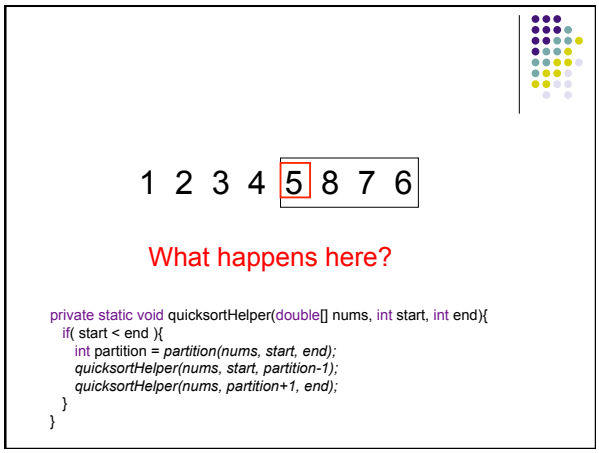
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What happens here?
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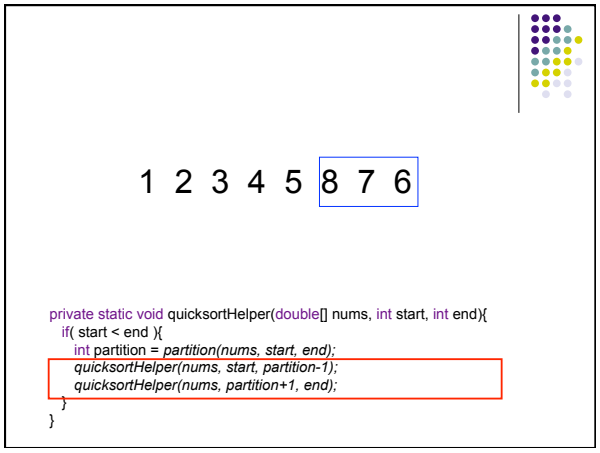
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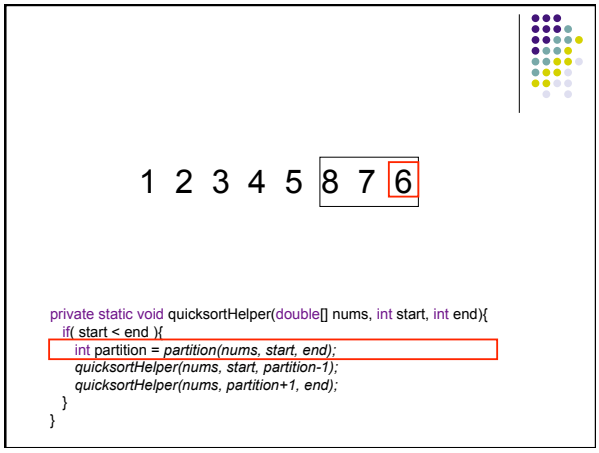
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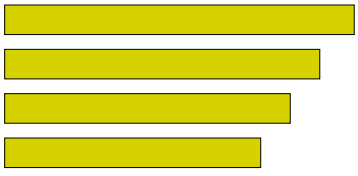
Running time of Quicksort?

Worst case?

Each call to Partition splits the array into an empty array and n-1 array



Quicksort: Worst case running time

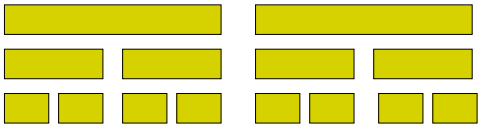
$n-1 + n-2 + n-3 + \ldots + 1 = O(n^2)$

When does this happen?

- sorted
- reverse sorted
- near sorted/reverse sorted

Quicksort best case?

Each call to Partition splits the array into two equal parts



…

How much work is done at each "level", i.e. running time of a level?

O(n)

Quicksort best case?

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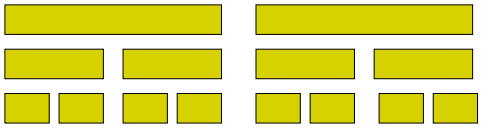

…

How many levels are there?

Similar to binary search, each call to Partition will throw away half the data until we're down to one element: $\log_2 n$ levels

Quicksort best case?

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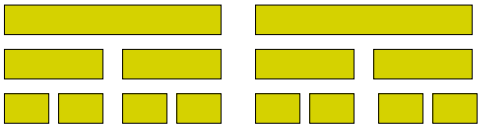


…

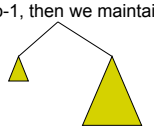
Overall runtime?

O(n log n)

Quicksort Average case?

Two intuitions

- As long as the Partition procedure always splits the array into some constant ratio between the left and the right, say L-to-R, e.g. 9-to-1, then we maintain O(n log n)



- As long as we only have a constant number of "bad" partitions intermixed with a "good partition" then we maintain O(n log n)

How can we avoid the worst case?

Inject randomness into the data

```
private static void randomizedPartition(double[] nums, int start, int end){
   int i = random(start, end);
   swap(nums, i, end);
   return partition = partition(nums, start, end);
}
```

Randomized quicksort is average case O(n log n)

What is the wost case running time of randomized Quicksort?

$O(n^2)$

We could still get very unlucky and pick “bad” partitions at every step