ADVANCED PERCEPTRON LEARNING

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CS 451 – Fall 2013

Admin

Assignment 2

- contest ☹
- due Sunday night!

Linear models

A linear model in *n*-dimensional space (i.e. *n* features) is define by *n*+1 weights:

In two dimensions, a line:

$$0 = w_1 f_1 + w_2 f_2 + b$$ (where b = -a)

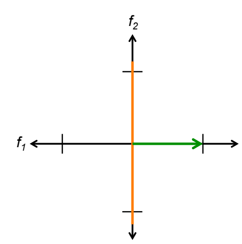
In three dimensions, a plane:

$$0 = w_1 f_1 + w_2 f_2 + w_3 f_3 + b$$

In *n*-dimensions, a *hyperplane*

$$0 = b + \sum_{i=1}^{n} w_i f_i$$

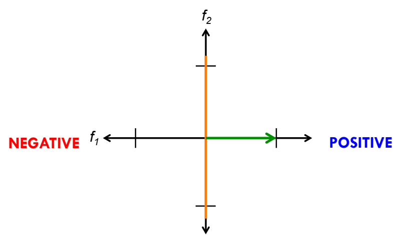
Learning a linear classifier



What does this model currently say?

w=(1,0)

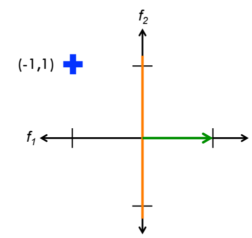
Learning a linear classifier



w=(1,0)

Learning a linear classifier

$0 = w_1 f_1 + w_2 f_2$



Is our current guess:
right or wrong?

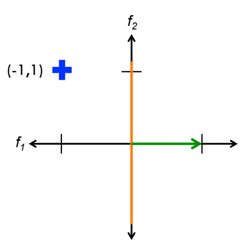
w=(1,0)

Learning a linear classifier

$0 = w_1 f_1 + w_2 f_2$

$1 * f_1 + 0 * f_2 =$

$1 * -1 + 0 * 1 = -1$



predicts negative, wrong

How should we update the model?

w=(1,0)

A closer look at why we got it wrong

$w_1 \quad w_2$

(-1, 1, positive)

$1 * f_1 + 0 * f_2 =$

$1 * -1 + 0 * 1 = -1$

We'd like this value to be positive since it's a positive value

contributed in the wrong direction

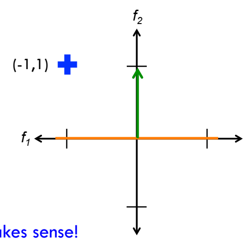
could have contributed (positive feature), but didn't

decrease
1 -> 0

increase
0 -> 1

Learning a linear classifier

$$0 = w_1 f_1 + w_2 f_2$$

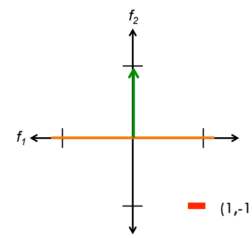


Graphically, this also makes sense!

w=(0,1)

Learning a linear classifier

$$0 = w_1 f_1 + w_2 f_2$$



Is our current guess: right or wrong?

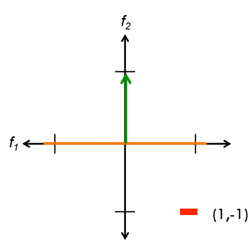
w=(0,1)

Learning a linear classifier

$$0 = w_1 f_1 + w_2 f_2$$

$$0 * f_1 + 1 * f_2 =$$

$$0 * 1 + 1 * -1 = -1$$



predicts negative, correct

How should we update the model?

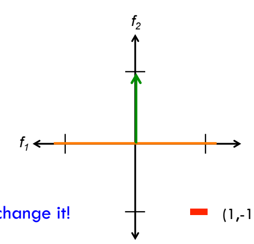
w=(0,1)

Learning a linear classifier

$$0 = w_1 f_1 + w_2 f_2$$

$$0 * f_1 + 1 * f_2 =$$

$$0 * 1 + 1 * -1 = -1$$

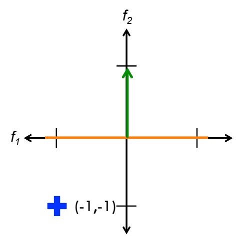


Already correct… don't change it!

w=(0,1)

Learning a linear classifier

$$0 = w_1 f_1 + w_2 f_2$$



Is our current guess: right or wrong?

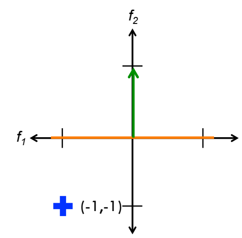
w=(0,1)

Learning a linear classifier

$$0 = w_1 f_1 + w_2 f_2$$

$$0 * f_1 + 1 * f_2 =$$

$$0 * -1 + 1 * -1 = -1$$



predicts negative, wrong

How should we update the model?

w=(0,1)

A closer look at why we got it wrong

w_1 w_2

(-1, -1, positive)

$$0 * f_1 + 1 * f_2 =$$

$$0 * -1 + 1 * -1 = -1$$

We'd like this value to be positive since it's a positive value

didn't contribute, but could have

contributed in the wrong direction

decrease

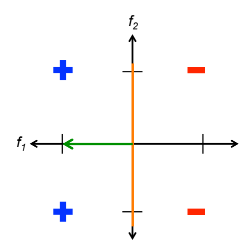
0 -> -1

decrease

1 -> 0

Learning a linear classifier

f_1, f_2, *label*
-1,-1, positive
-1, 1, positive
1, 1, negative
1,-1, negative



w=(-1,0)

Perceptron learning algorithm

repeat until convergence (or for some # of iterations):
  for each training example (f_1, f_2, …, f_n, label):
    check if it's correct based on the current model

    if not correct, update all the weights: $w_i f_i$
      if label positive and feature positive:
        increase weight (increase weight = predict more positive)
      if label positive and feature negative:
        decrease weight (decrease weight = predict more positive)
      if label negative and feature positive:
        decrease weight (decrease weight = predict more negative)
      if label negative and negative weight:
        increase weight (increase weight = predict more negative)

A trick…

Let positive label = 1 and negative label = -1

	label * f_i
if label positive and feature positive: increase weight (increase weight = predict more positive)	1*1=1
if label positive and feature negative: decrease weight (decrease weight = predict more positive)	1*-1=-1
if label negative and feature positive: decrease weight (decrease weight = predict more negative)	-1*1=-1
if label negative and negative weight: increase weight (increase weight = predict more negative)	-1*-1=1

A trick…

Let positive label = 1 and negative label = -1

	label * f_i
if label positive and feature positive: increase weight (increase weight = predict more positive)	1*1=1
if label positive and feature negative: decrease weight (decrease weight = predict more positive)	1*-1=-1
if label negative and feature positive: decrease weight (decrease weight = predict more negative)	-1*1=-1
if label negative and negative weight: increase weight (increase weight = predict more negative)	-1*-1=1

Perceptron learning algorithm

repeat until convergence (or for some # of iterations):
  for each training example (f_1, f_2, …, f_n, label):
    check if it's correct based on the current model

    if not correct, update all the weights:
      for each w_i:
        $w_i = w_i + f_i$*label
      $b = b + $ label

How do we check if it's correct?

Perceptron learning algorithm

repeat until convergence (or for some # of iterations):

for each training example (f_1, f_2, …, f_n, label):

$$prediction = b + \sum_{i=1}^{n} w_i f_i$$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label

$b = b$ + label

Perceptron learning algorithm

repeat until convergence (or for some # of iterations):

for each training example (f_1, f_2, …, f_n, label):

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if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label

$b = b$ + label

Would this work for non-binary features, i.e. real-valued?

Your turn ☺

repeat until convergence (or for some # of iterations):

for each training example (f_1, f_2, …, f_n, label):

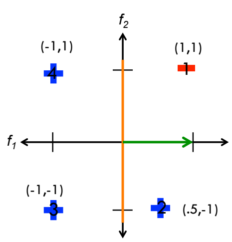
$$prediction = \sum_{i=1}^{n} w_i f_i$$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label

- Repeat until convergence
- Keep track of w_1, w_2 as they change
- Redraw the line after each step



w = (1, 0)

Your turn ☺

repeat until convergence (or for some # of iterations):

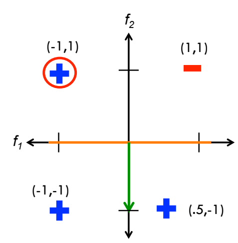
for each training example (f_1, f_2, …, f_n, label):

$$prediction = \sum_{i=1}^{n} w_i f_i$$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label



w = (0, -1)

Your turn ☺

repeat until convergence (or for some # of iterations):

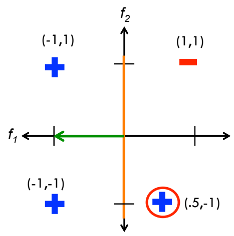
for each training example (f_1, f_2, …, f_n, label):

$prediction = \sum_{i=1}^{n} w_i f_i$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label



w = (-1, 0)

Your turn ☺

repeat until convergence (or for some # of iterations):

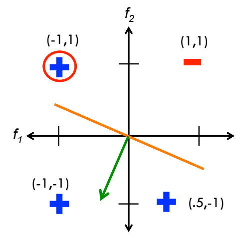
for each training example (f_1, f_2, …, f_n, label):

$prediction = \sum_{i=1}^{n} w_i f_i$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label



w = (-.5, -1)

Your turn ☺

repeat until convergence (or for some # of iterations):

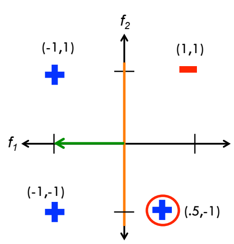
for each training example (f_1, f_2, …, f_n, label):

$prediction = \sum_{i=1}^{n} w_i f_i$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label



w = (-1.5, 0)

Your turn ☺

repeat until convergence (or for some # of iterations):

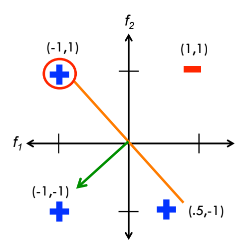
for each training example (f_1, f_2, …, f_n, label):

$prediction = \sum_{i=1}^{n} w_i f_i$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

$w_i = w_i + f_i$*label



w = (-1, -1)

Your turn ☺

repeat until convergence (or for some # of iterations):

for each training example (f_1, f_2, …, f_n, label):

$prediction = \sum_{i=1}^{n} w_i f_i$

if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

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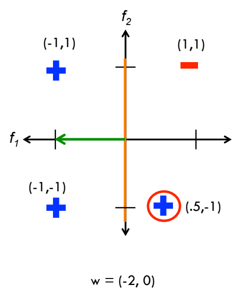


Your turn ☺

repeat until convergence (or for some # of iterations):

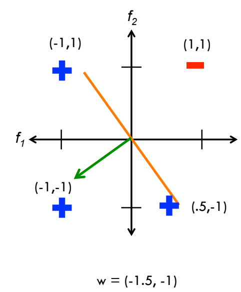
for each training example (f_1, f_2, …, f_n, label):

$prediction = \sum_{i=1}^{n} w_i f_i$

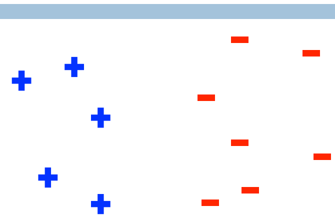
if *prediction* * *label* ≤ 0: // they don't agree

for each w_i:

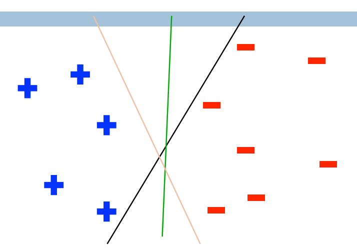
$w_i = w_i + f_i$*label



Which line will it find?



Which line will it find?



Only guaranteed to find ***some*** line that separates the data

Convergence

repeat until convergence (or for some # of iterations):

  for each training example ($f_1, f_2, \ldots, f_n$, label):

  $prediction = b + \sum_{i=1}^{n} w_i f_i$

  if *prediction* * *label* ≤ 0: // they don't agree

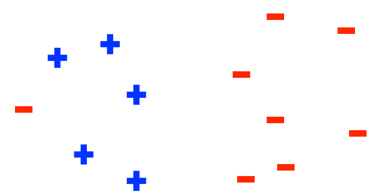
   for each w_i:

    $w_i = w_i + f_i$*label

   b = b + label

Why do we also have the "some # iterations" check?

Handling non-separable data



If we ran the algorithm on this it would never converge!

Convergence

repeat until convergence (or for some # of iterations):

  for each training example ($f_1, f_2, \ldots, f_n$, label):

  $prediction = b + \sum_{i=1}^{n} w_i f_i$

  if *prediction* * *label* ≤ 0: // they don't agree

   for each w_i:

    $w_i = w_i + f_i$*label

   b = b + label

Also helps avoid overfitting!
(This is harder to see in 2-D examples, though)

Ordering

repeat until convergence (or for some # of iterations):

  for each training example ($f_1, f_2, \ldots, f_n$, label):

  $prediction = b + \sum_{i=1}^{n} w_i f_i$

  if *prediction* * *label* ≤ 0: // they don't agree

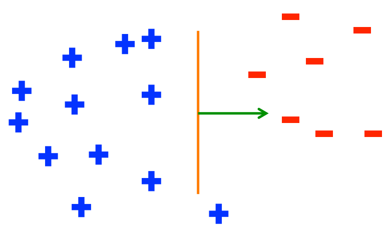
   for each w_i:

    $w_i = w_i + f_i$*label

   b = b + label

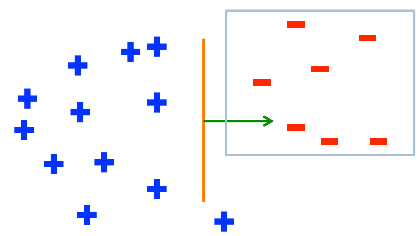
What order should we traverse the examples?
Does it matter?

Order matters

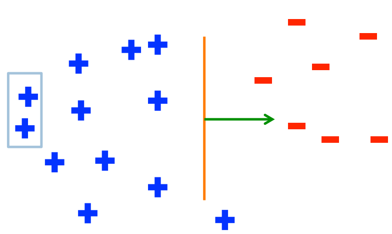


What would be a good/bad order?

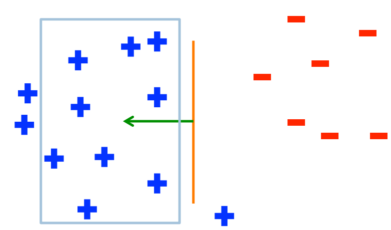
Order matters: a bad order



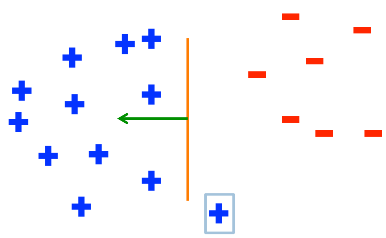
Order matters: a bad order



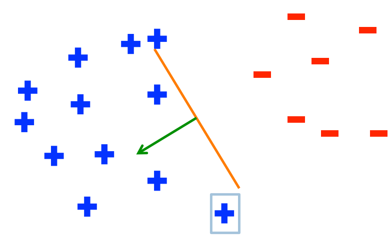
Order matters: a bad order



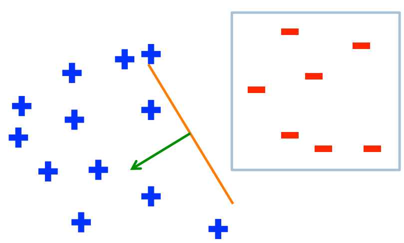
Order matters: a bad order



Order matters: a bad order



Order matters: a bad order



Solution?

Ordering

repeat until convergence (or for some # of iterations):

  randomize order or training examples

  for each training example (f_1, f_2, …, f_n, label):

$$prediction = b + \sum_{i=1}^{n} w_i f_i$$

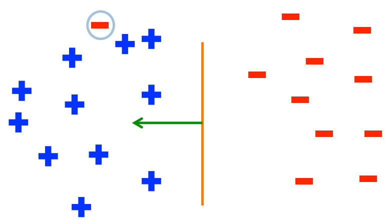
    if *prediction* * *label* $\leq$ 0: // they don't agree

      for each w_i:

        w_i = w_i + f_i*label

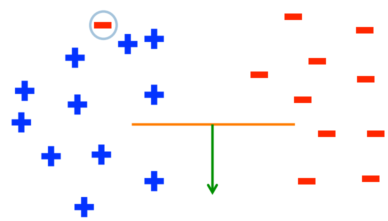
      b = b + label

Improvements



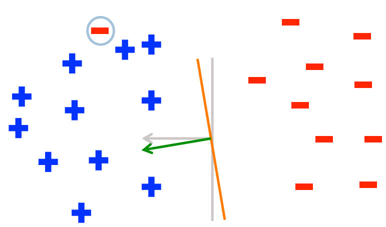
What will happen when we examine this example?

Improvements



Does this make sense? What if we had previously gone through ALL of the other examples correctly?

Improvements



Maybe just move it slightly in the direction of correction

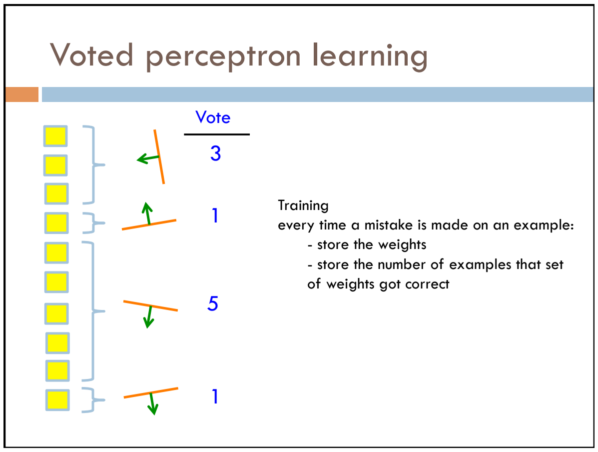
Voted perceptron learning

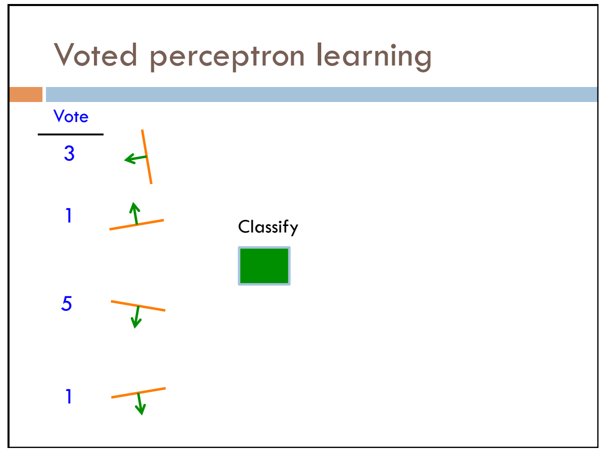
Training

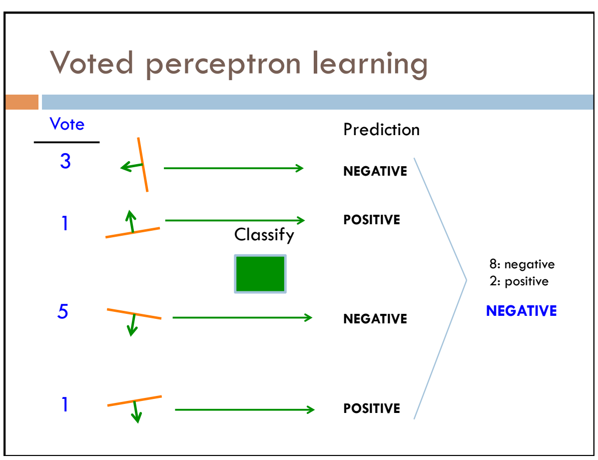
- every time a mistake is made on an example:
  - store the weights (i.e. before changing for current example)
  - store the number of examples that set of weights got correct

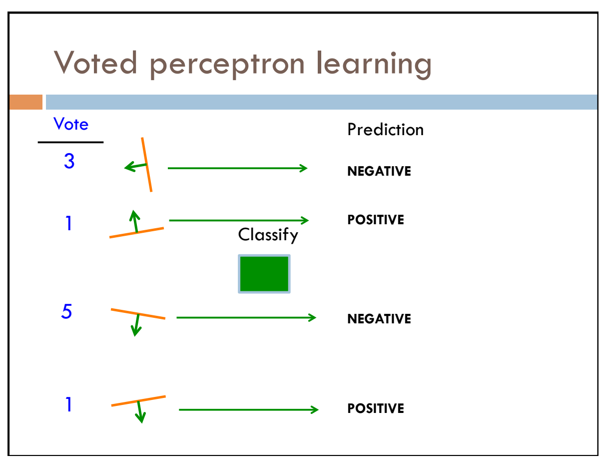
Classify

- calculate the prediction from ALL saved weights
- multiply each prediction by the number it got correct (i.e a weighted vote) and take the sum over all predictions
- said another way: pick whichever prediction has the most votes


Voted perceptron learning
Vote
3
1
5
1
Training
every time a mistake is made on an example:
- store the weights
- store the number of examples that set of weights got correct


Voted perceptron learning
Vote
3
1
5
1
Classify


Voted perceptron learning
Vote
3
1
5
1
Prediction
NEGATIVE
POSITIVE
NEGATIVE
POSITIVE
Classify
8: negative
2: positive
NEGATIVE


Voted perceptron learning
Vote
3
1
5
1
Prediction
NEGATIVE
POSITIVE
NEGATIVE
POSITIVE
Classify

Voted perceptron learning

Works much better in practice

Avoids overfitting, though it can still happen

Avoids big changes in the result by examples examined at the end of training

Voted perceptron learning

Training
- every time a mistake is made on an example:
  - store the weights (i.e. before changing for current example)
  - store the number of examples that set of weights got correct

Classify
- calculate the prediction from ALL saved weights
- multiply each prediction by the number it got correct (i.e a weighted vote) and take the sum over all predictions
- said another way: pick whichever prediction has the most votes

Any issues/concerns?

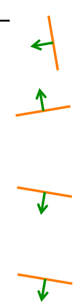
Voted perceptron learning

Training
- every time a mistake is made on an example:
  - store the weights (i.e. before changing for current example)
  - store the number of examples that set of weights got correct

Classify
- calculate the prediction from ALL saved weights
- multiply each prediction by the number it got correct (i.e a weighted vote) and take the sum over all predictions
- said another way: pick whichever prediction has the most votes

1. Can require a lot of storage
2. Classifying becomes very, very expensive

Average perceptron



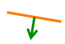
Vote		
3		$w_1^1, w_2^1, \ldots, w_n^1, b^1$
1		$w_1^2, w_2^2, \ldots, w_n^2, b^2$
5		$w_1^3, w_2^3, \ldots, w_n^3, b^3$
1		$w_1^4, w_2^4, \ldots, w_n^4, b^4$

$$\overline{w_i} = \frac{3w_i^1 + 1w_i^2 + 5w_i^3 + 1w_i^4}{10}$$

The final weights are the *weighted average* of the previous weights

How does this help us?

Average perceptron

Vote		
3		$w_1^1, w_2^1, \ldots, w_n^1, b^1$
1		$w_1^2, w_2^2, \ldots, w_n^2, b^2$
5		$w_1^3, w_2^3, \ldots, w_n^3, b^3$
1		$w_1^4, w_2^4, \ldots, w_n^4, b^4$

$$\overline{w_i} = \frac{3w_i^1 + 1w_i^2 + 5w_i^3 + 1w_i^4}{10}$$

The final weights are the *weighted average* of the previous weights

Can just keep a running average!

Perceptron learning algorithm

repeat until convergence (or for some # of iterations):

  for each training example (f_1, f_2, …, f_n, label):

   $prediction = b + \sum_{i=1}^{n} w_i f_i$

   if *prediction* * *label* ≤ 0: // they don't agree

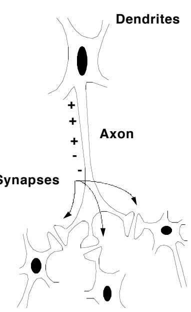
    for each w_i:

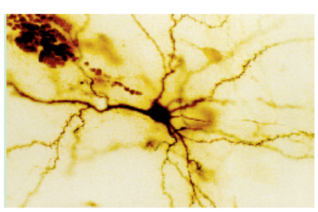
     $w_i = w_i + f_i$*label

    b = b + label

Why is it called the "perceptron" learning algorithm if what it learns is a line? Why not "line learning" algorithm?

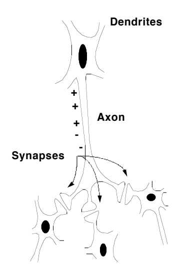
Our Nervous System





Neuron

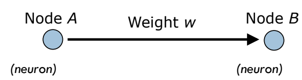
Our nervous system: *the computer science view*



the human brain is a large collection of interconnected neurons

a NEURON is a brain cell

- collect, process, and disseminate electrical signals
- Neurons are connected via synapses
- They FIRE depending on the conditions of the neighboring neurons



w is the strength of signal sent between A and B.

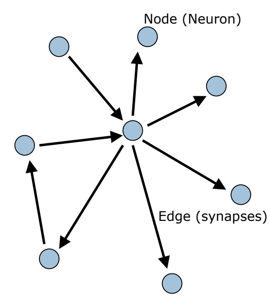
If *A* fires and *w* is **positive**, then *A* **stimulates** *B*.

If *A fires* and *w* is **negative**, then *A* **inhibits** *B*.

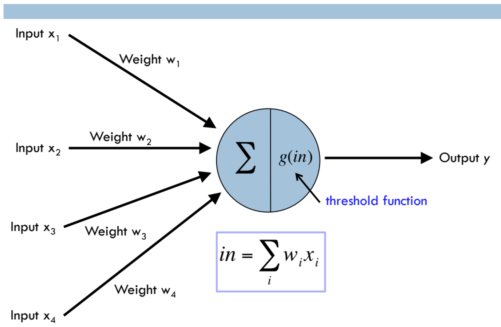
If a node is stimulated enough, then it also fires.

How much stimulation is required is determined by its **threshold**.

Neural Networks



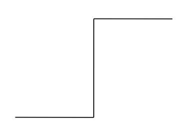
A Single Neuron/Perceptron



Possible threshold functions

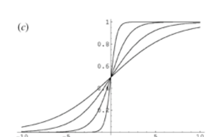
hard threshold:

if *in* (the sum of weights) >= *threshold* 1, 0 otherwise

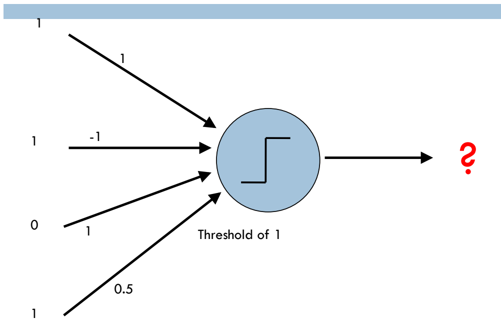


Sigmoid

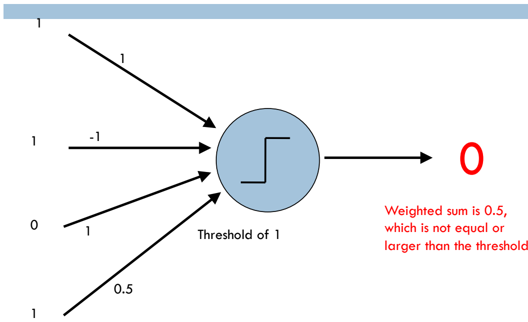
$$g(x) = \frac{1}{1+e^{-ax}}$$



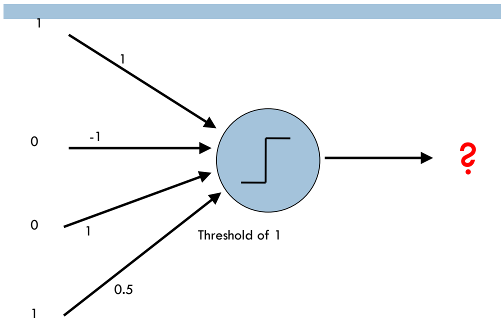
A Single Neuron/Perceptron



A Single Neuron/Perceptron



A Single Neuron/Perceptron



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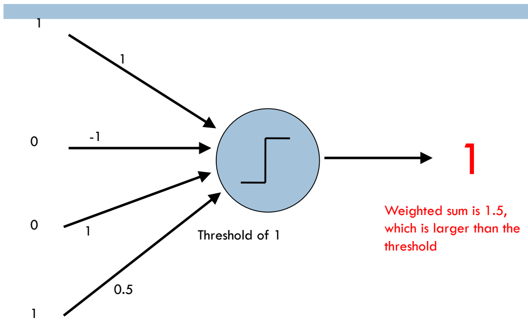
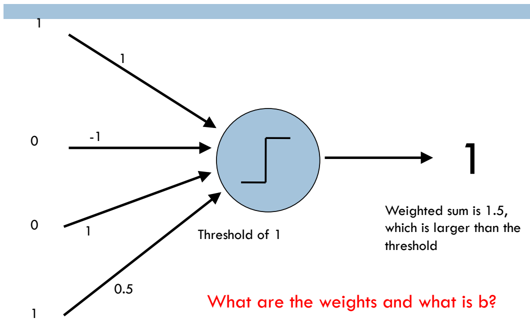


A Single Neuron/Perceptron



What are the weights and what is b?

History of Neural Networks

McCulloch and Pitts (1943) – introduced model of artificial neurons and suggested they could learn

Hebb (1949) – Simple updating rule for learning

Rosenblatt (1962) - the *perceptron* model

Minsky and Papert (1969) – wrote *Perceptrons*

Bryson and Ho (1969, but largely ignored until 1980s) – invented back-propagation learning for multilayer networks