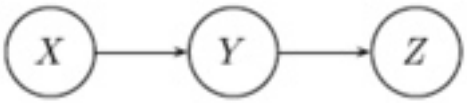
CS151 - Written Problem 4
To be done by: Monday, Oct. 4

1. Exercise 13.8
2. Exercise 13.21

   The next two problems are taken from
   http://www-nlp.stanford.edu/~grenager/cs121//handouts/hw2.pdf

3. In this problem were going to prove the conditional independence properties of the following Bayesian network:



   (a) What are the conditional probability distributions (CPDs) that are represented in this Bayesian network?

   (b) Write down the joint probability distribution over X, Y , and Z as represented by this Bayesian network. This expression should be written in terms of the CPDs you enumerated in a. (plus any unconditional distributions).

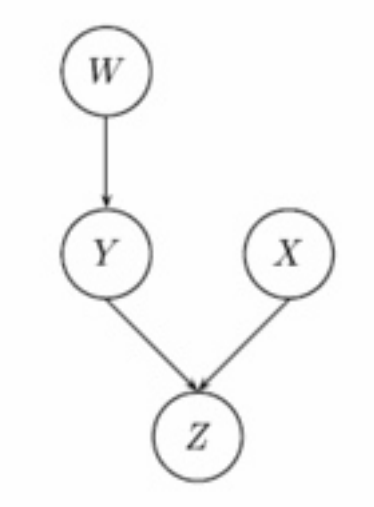
   (c) Now write down an expression in terms of these for $P(X, Z)$, the marginal probability of X and Z (hint: sum the variable Y "out" from the joint distribution you wrote above).

   (d) Based on the expression in c., and the definition of independence, are X and Z independent?

   (e) Write down an expression for $P(X, Z|Y)$, again in terms of these simplified probability distributions

(f) Based on this expression, and the definition of conditional independence, are X and Z conditionally independent given Y?

4. In this question we examine the conditional independence assumptions encoded in the Bayesian network graph topology. Consider the following Bayesian network:



(a) Write down all the independencies not conditioned on other variables that are enforced by this Bayesian network, using the notation $A \perp\!\!\!\perp B$ to mean that A is independent of B.

(b) Write down three independencies which do not necessarily hold in this Bayesian network.

(c) Write down all the conditional independencies that are enforced by this Bayesian network, using the notation $A \perp\!\!\!\perp B|C$ to mean that A is conditionally independent of B given C.

(d) Write down three conditional independencies which do not necessarily hold in this Bayesian network.