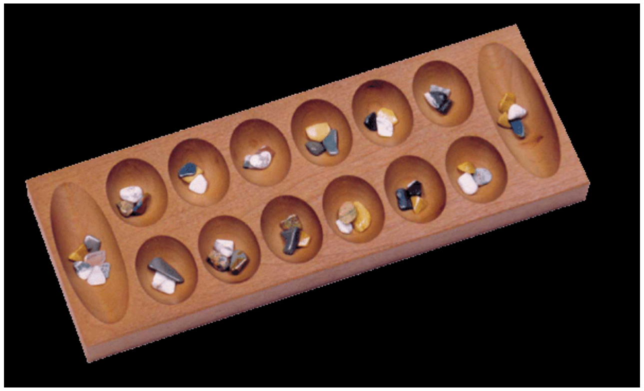
http://www.youtube.com/watch?v=ICgL1OWsn58

Adversarial Search

CS151
David Kauchak
Fall 2010

Some material borrowed from :
Sara Owsley Sood and others

Admin



- Reading?
- Assignment 2
  - On the web page
  - 3 parts
  - Anyone looking for a partner?
  - Get started!
- Written assignments
  - Post solutions to W1 today
  - Post next written assignment soon

A quick review of search

- Rational thinking via search – determine a plan of actions by searching from starting state to goal state
- Uninformed search vs. informed search
  - what's the difference?
  - what are the techniques we've seen?
  - pluses and minuses?
- Heuristic design
  - admissible?
  - dominant?

Why should we study games?

- Clear success criteria
- Important historically for AI
- Fun ☺
- Good application of search
  - hard problems (chess 35^{100} nodes in search tree, 10^{40} legal states)
- Some real-world problems fit this model
  - game theory (economics)
  - multi-agent problems

Types of games

What are some of the games you've played?

Types of games: game properties

- single-player vs. 2-player vs. multiplayer
- Fully observable (perfect information) vs. partially observable
- Discrete vs. continuous
- real-time vs. turn-based
- deterministic vs. non-deterministic (chance)

Strategic thinking $\stackrel{?}{=}$ intelligence

For reasons previously stated, two-player games have been a focus of AI since its inception…

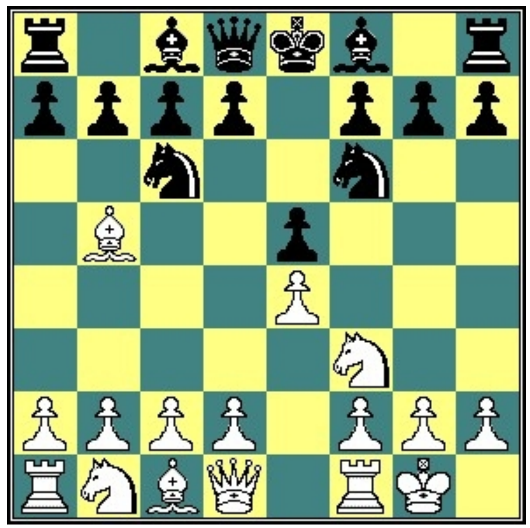
Begs the question: Is strategic thinking the same as intelligence?

Strategic thinking $\overset{?}{=}$ intelligence

Humans and computers have different relative strengths in these games:

humans

good at evaluating the strength of a board for a player



computers

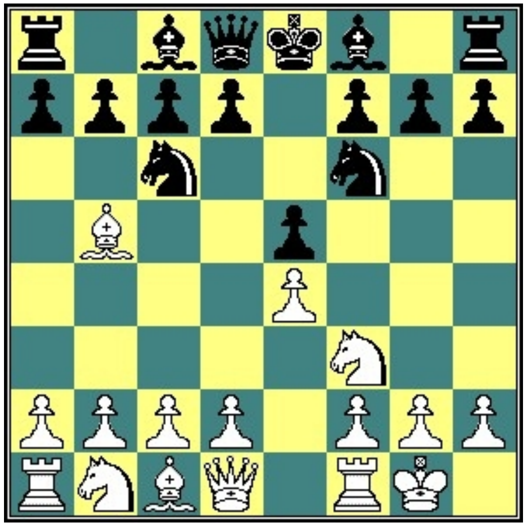
good at looking ahead in the game to find winning combinations of moves

Strategic thinking $\stackrel{?}{=}$ intelligence

How could you figure out how humans approach playing chess?

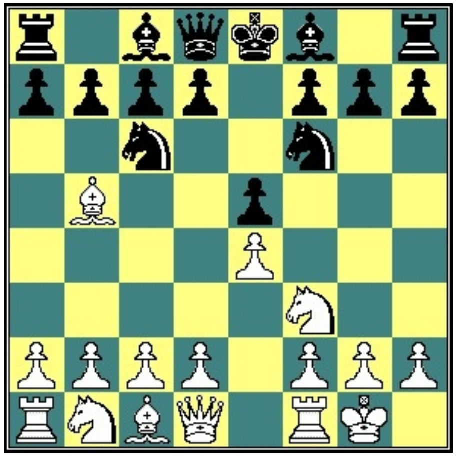
humans

good at evaluating the strength of a board for a player



How humans play games…

An experiment (by deGroot) was performed in which chess positions were shown to novice and expert players…



- experts could reconstruct these perfectly
- novice players did far worse…

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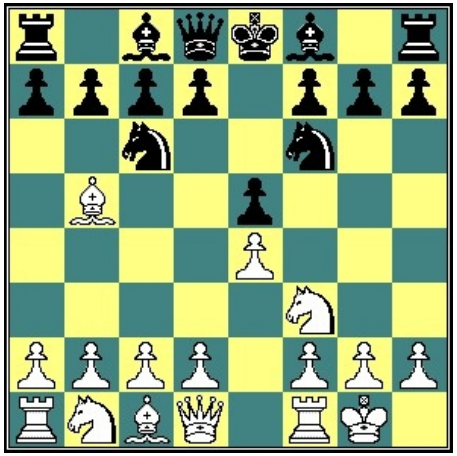
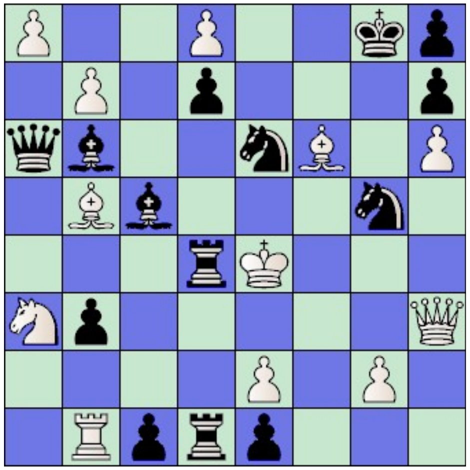


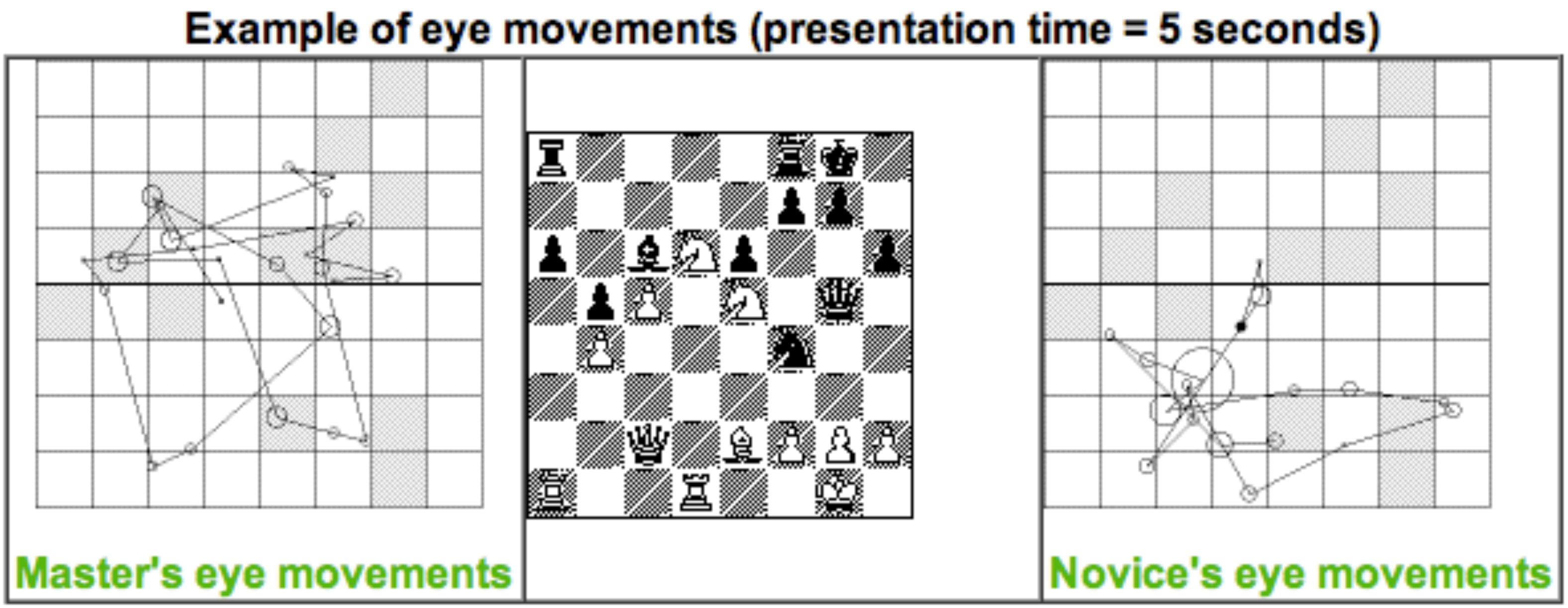
- experts could reconstruct these perfectly
- novice players did far worse…

<u>Random</u> chess positions (not legal ones) were then shown to the two groups



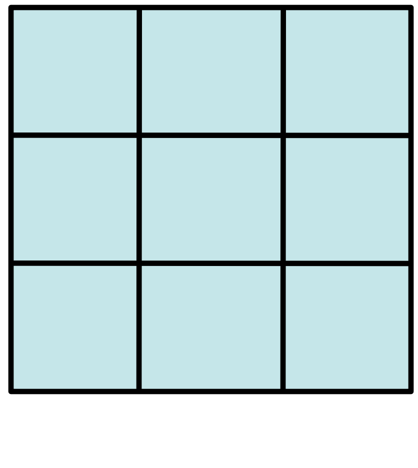
- experts and novices did just as badly at reconstructing them!

People are still working on this problem…



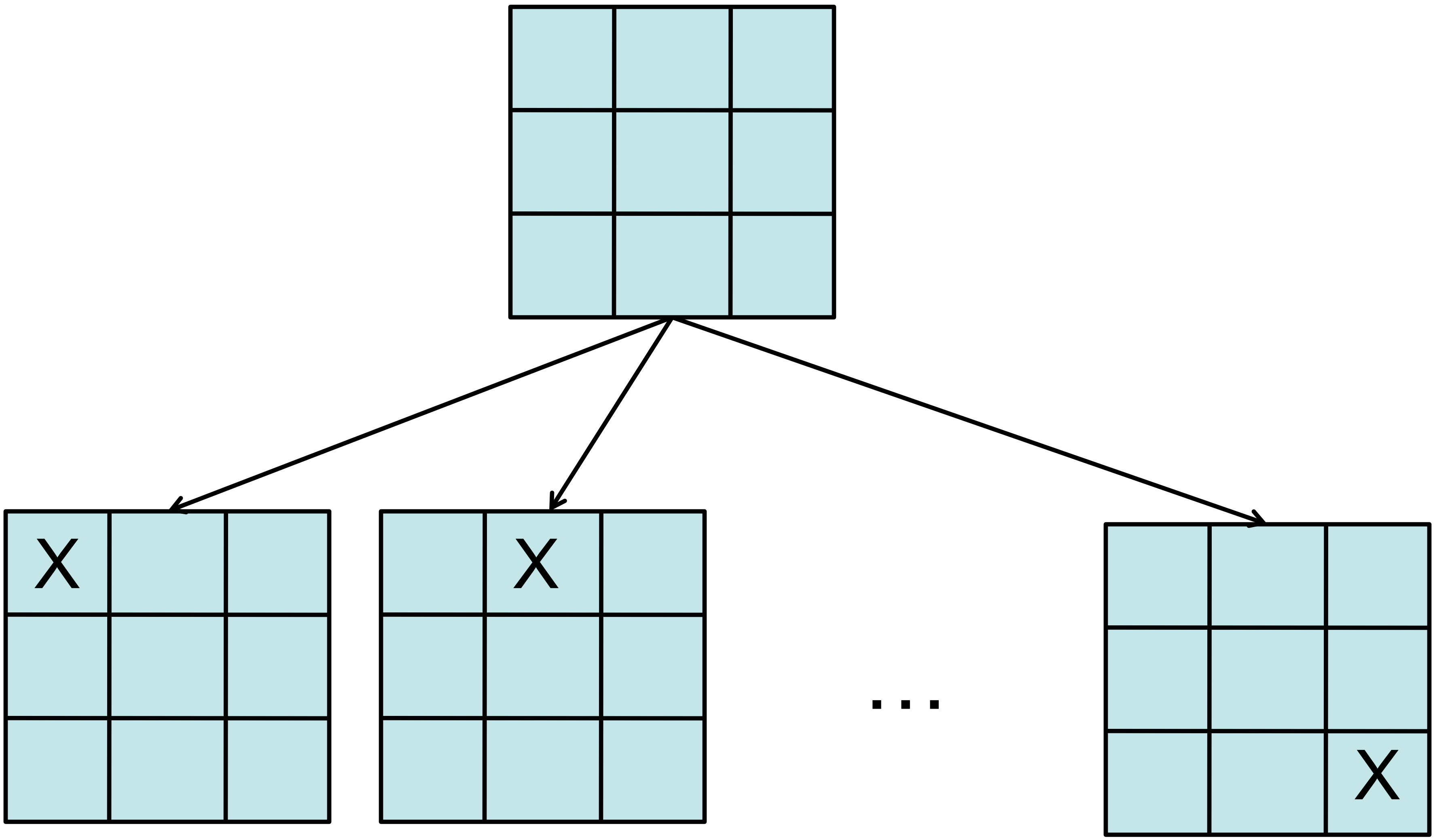
http://people.brunel.ac.uk/~hsstffg/frg-research/chess_expertise/

Tic Tac Toe as search



How can we pose this as a search problem?

Tic Tac Toe as search



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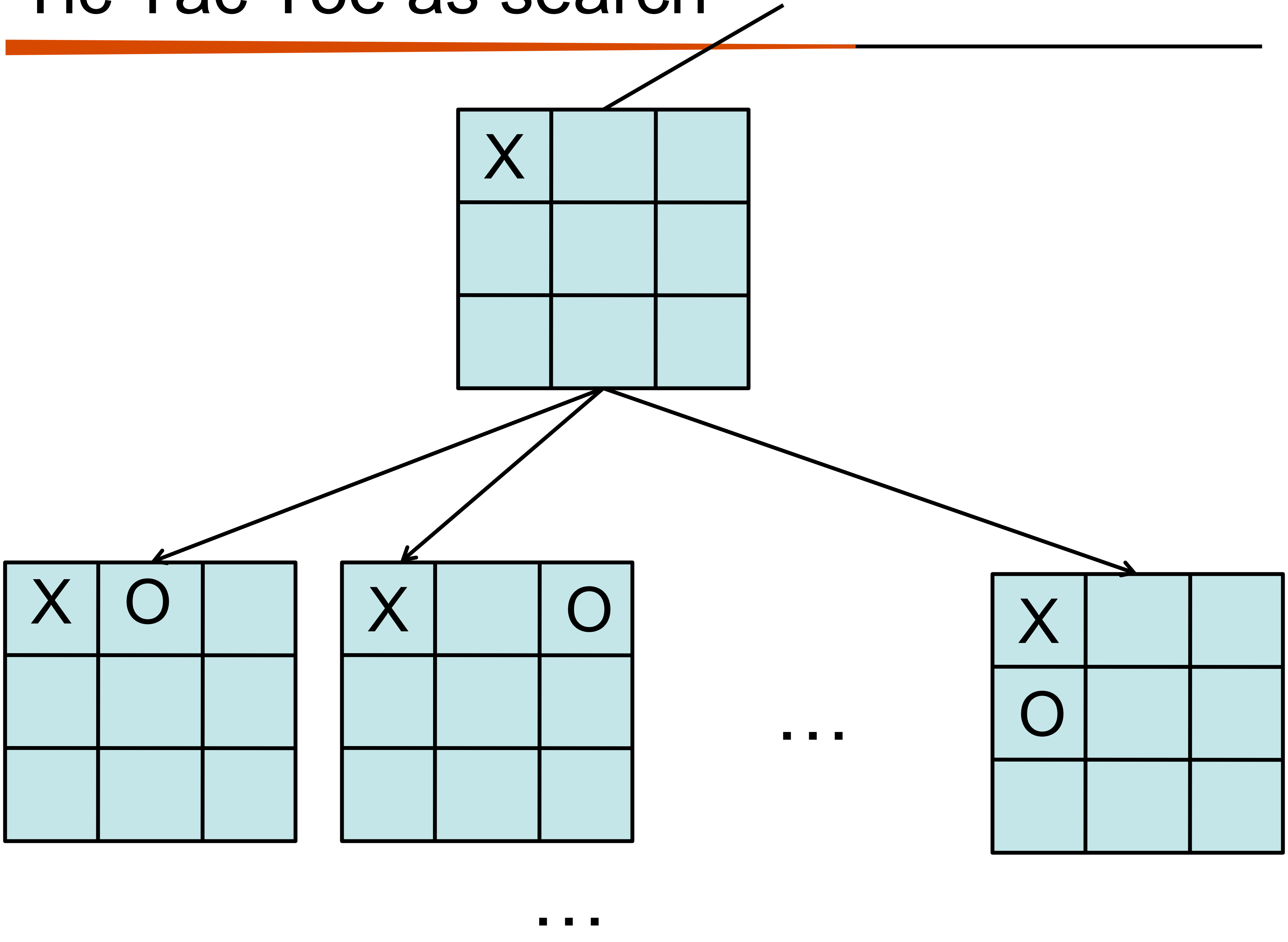


Tic Tac Toe as search

Eventually, we'll get to a leaf

X	X	O
X	O	O
X	O	X

+1

X	X	O
O	X	O
X	O	X

0

…

X	X	O
O	X	O
X		O

-1

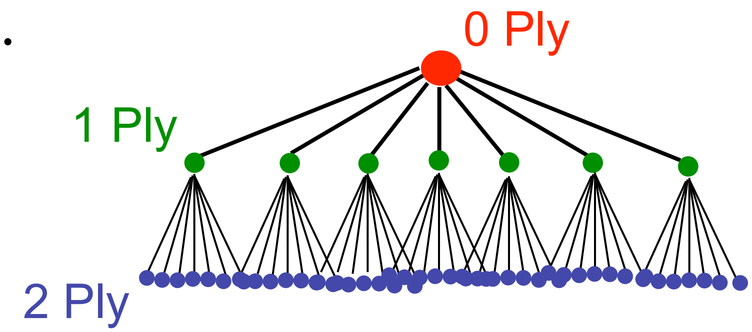
The **UTILITY** of a state tells us how good the states are.

Defining the problem

- INITIAL STATE – board position and the player whose turn it is
- SUCCESSOR FUNCTION– returns a list of (move, next state) pairs
- TERMINAL TEST – is game over? Are we in a terminal state?
- UTILITY FUNCTION – (objective or payoff func) gives a numeric value for terminal states (ie – chess – win/lose/draw +1/-1/0, backgammon +192 to -192)

Games' Branching Factors

• On average, there are ~35 possible moves that a chess player can make from any board configuration…



18 Ply!!

Hydra at home in the United Arab Emirates…

Branching Factor Estimates
for different two-player games

Tic-tac-toe	4
Connect Four	7
Checkers	10
Othello	30
Chess	40
Go	300

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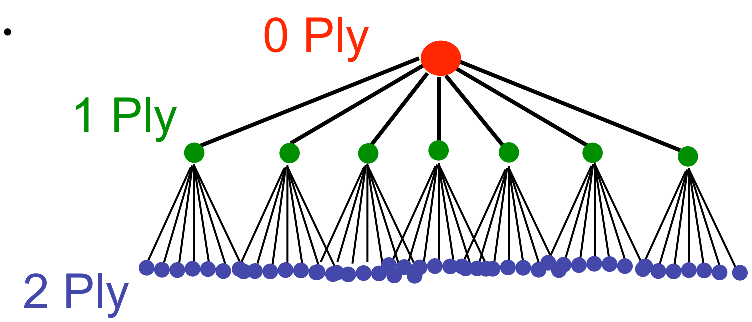

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Boundaries for qualitatively different games…

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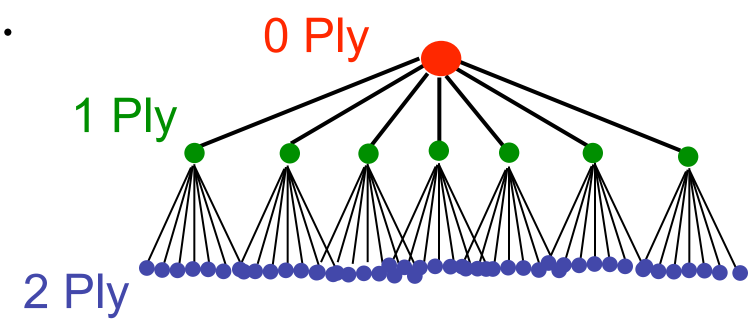


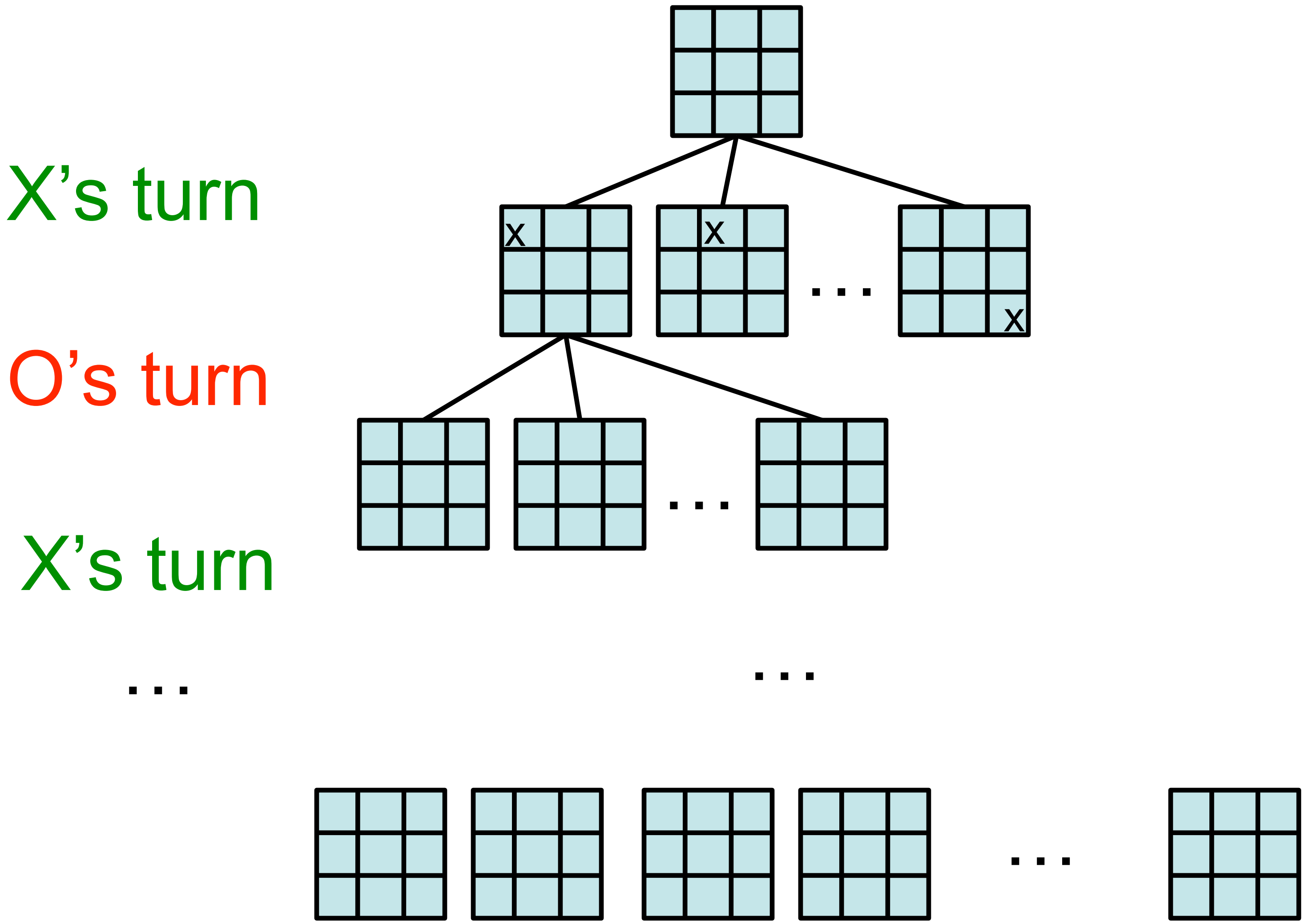
Branching Factor Estimates
for different two-player games

	Game	Branching factor
"solved" games	Tic-tac-toe	4
	Connect Four	7
CHINOOK (2007) →	Checkers	10
computer-dominated	Othello	30
	Chess	40
human-dominated	Go	300

Games vs. search problems?

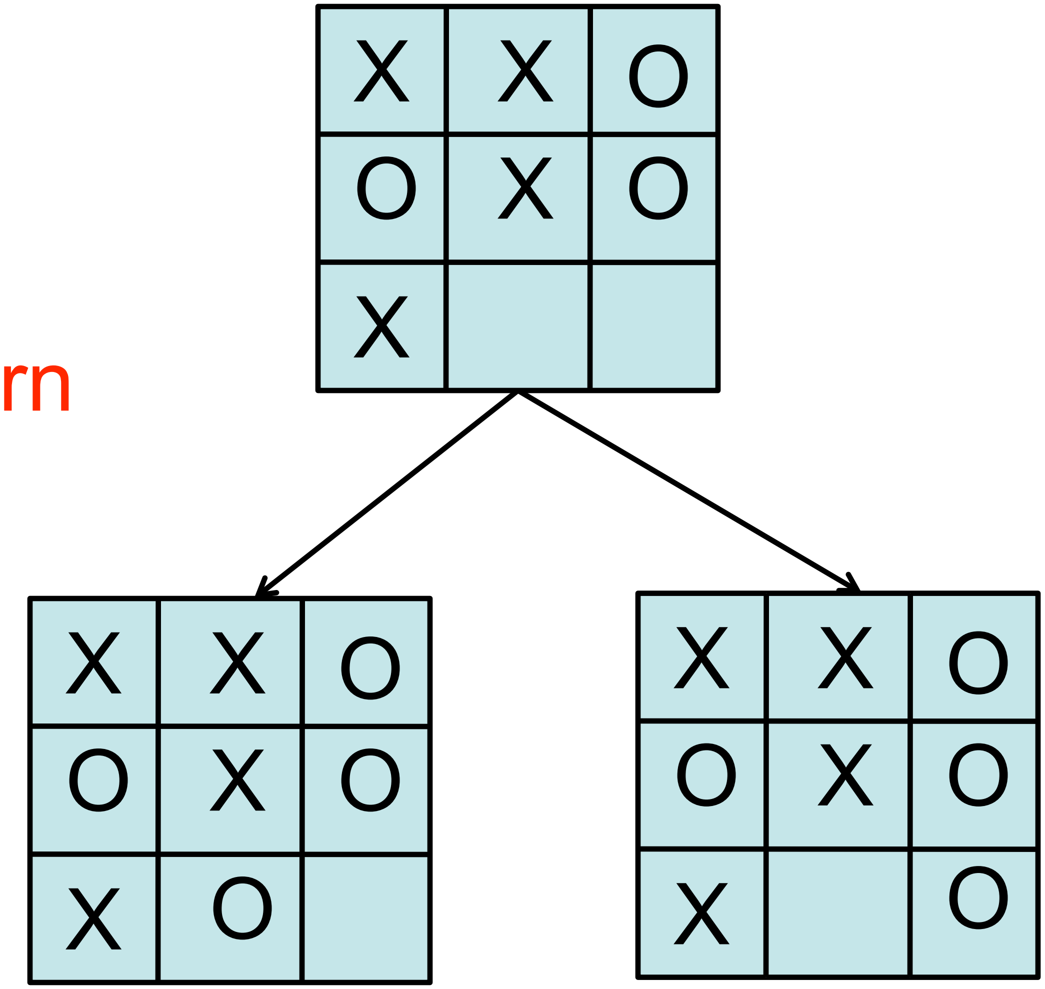
- Opponent!
  - unpredictable/uncertainty
  - deal with opponent strategy
- Time limitations
  - must make a move in a reasonable amount of time
  - can’t always look to the end
- Path costs
  - not about moves, but about UTILITY of the resulting state/winning

Back to Tic Tac TOe



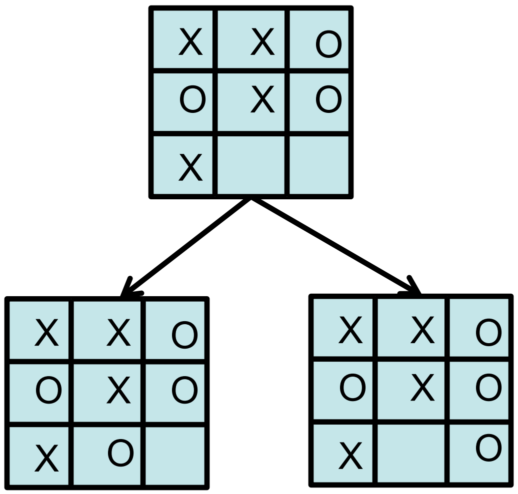
I’m X, what will ‘O’ do?

O’s turn



Minimizing risk

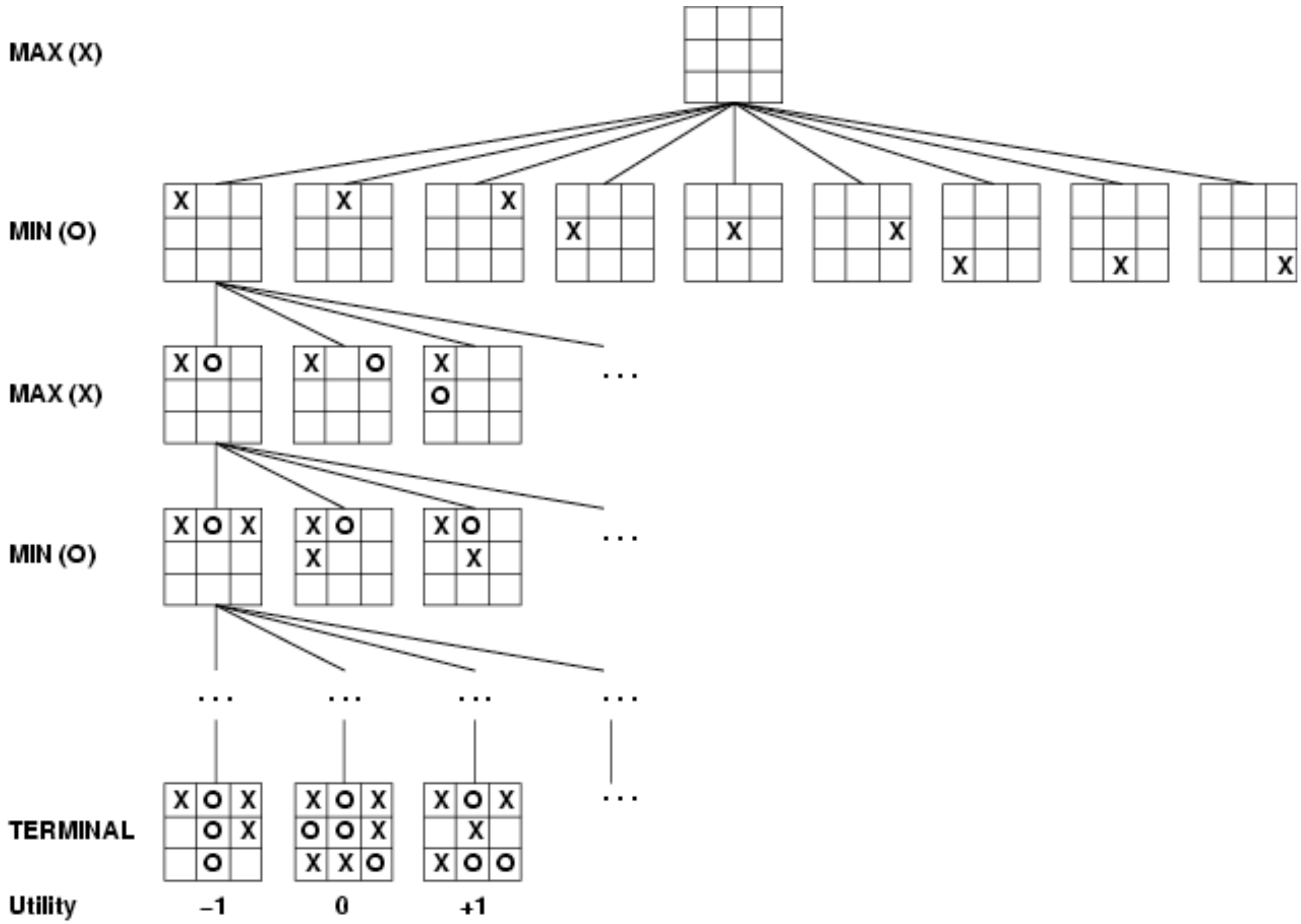
- The computer doesn't know what move O (the opponent) will make
- It can assume, though, that it will try and make the best move possible
- Even if O atually makes a different move, we're no worse off



Optimal Strategy

- An Optimal Strategy is one that is at least as good as any other, no matter what the opponent does
  - If there's a way to force the win, it will
  - Will only lose if there's no other option

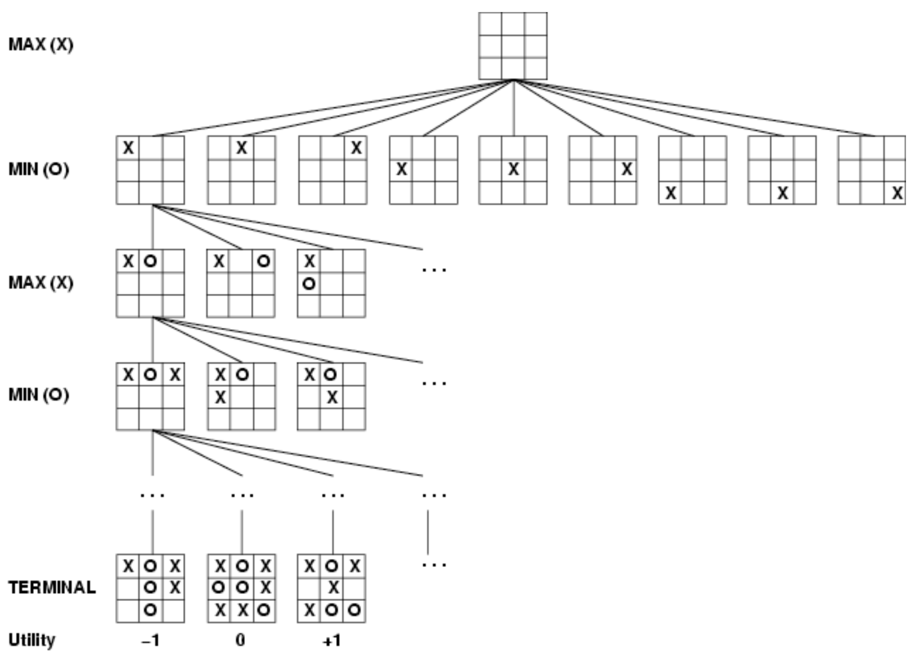
How can X play optimally?



How can X play optimally?

- Start from the leaves and propagate the utility up:
  – if X's turn, pick the move that maximizes the utility
  – if O's turn, pick the move that minimizes the utility

Is this optimal?



Minimax Algorithm: An Optimal Strategy

minimax(state) =
- if state is a terminal state
  Utility(state)
- if MAX's turn
  return the *maximum* of minimax(...)
  on all successors of current state
- if MIN's turn
  return the *minimum* of minimax(…)
  on all successors to current state

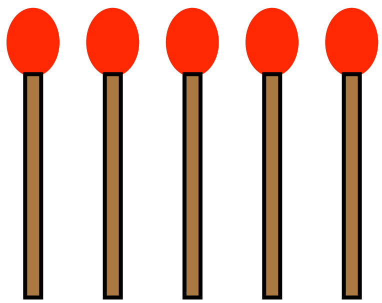
- Uses recursion to compute the "value" of each state
- Proceeds to the leaves, then the values are "backed up" through the tree as the recursion unwinds
- What type of search is this?
- What does this assume about how MIN will play? What if this isn't true?

```
def minimax(state):
  for all actions a in actions(state):
    return the a with the largest minValue(result(state,a))

def maxValue(state):
  if state is terminal:
    return utility(state)
  else:
    # return the a with the largest minValue(result(state,a))
    value = -∞
    for all actions a in actions(state):
      value = max(value, minValue(result(state,a))
    return value

def minValue(state):
  if state is terminal:
    return utility(state)
  else:
    # return the a with the smallest maxValue(result(state,a))
    value = +∞
    for all actions a in actions(state):
      value = min(value, maxValue(result(state,a))
    return value
```

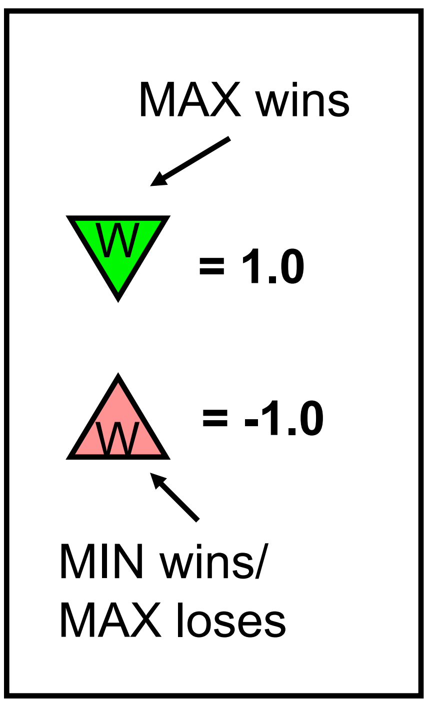
ME: Assume the opponent will try and minimize value, maximize my move

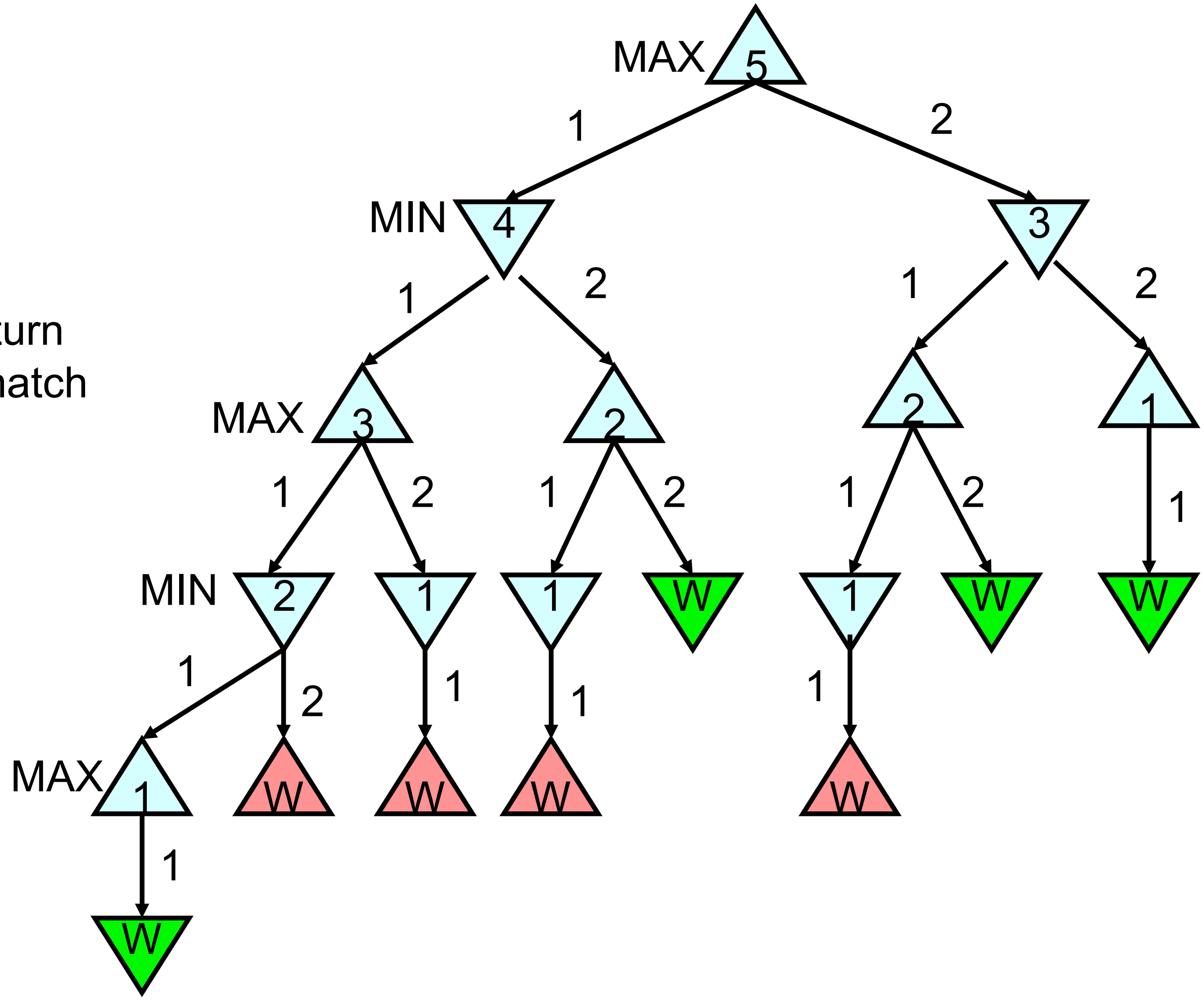
OPPONENT: Assume I will try and maximize my value, minimize his/her move

Baby Nim



Take 1 or 2 at each turn
Goal: take the last match





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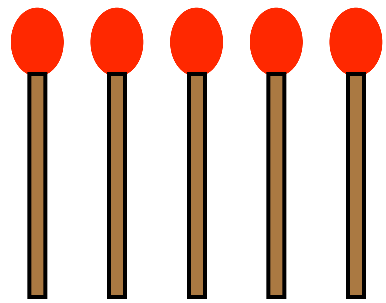

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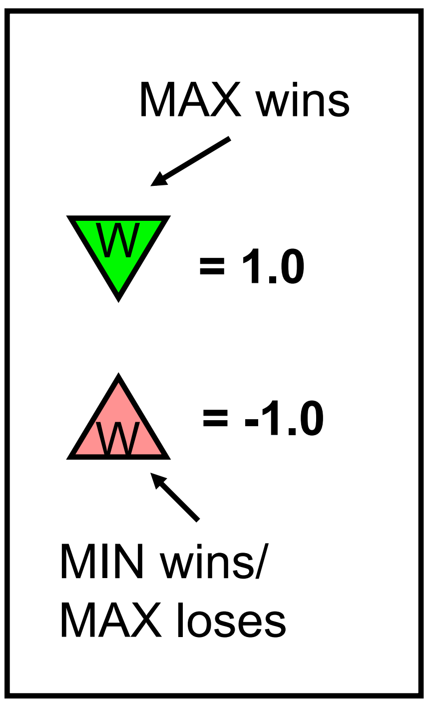

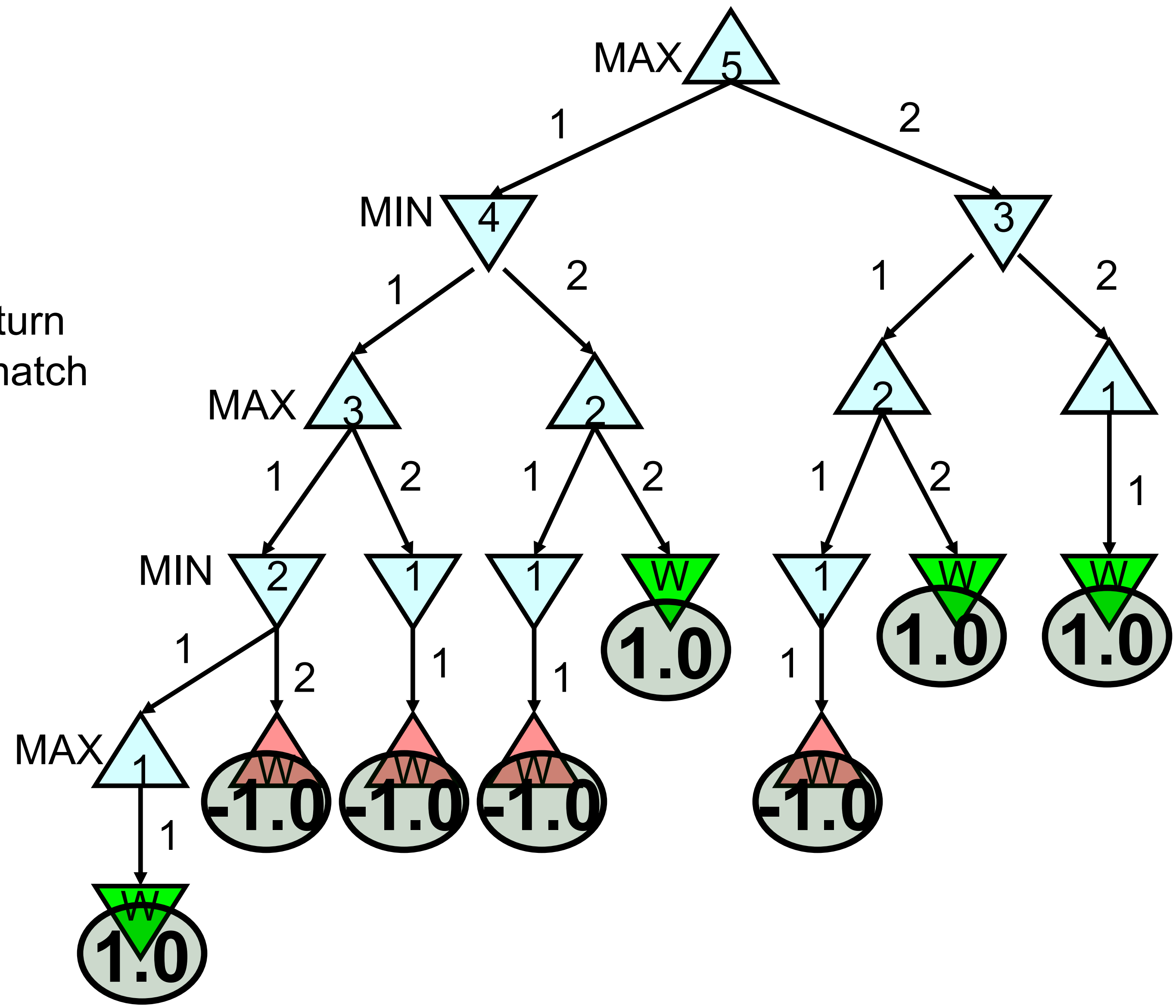

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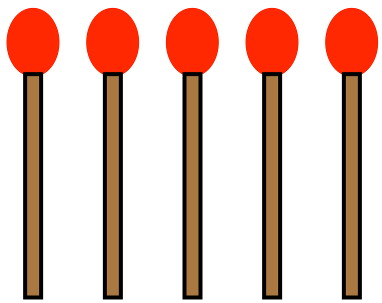

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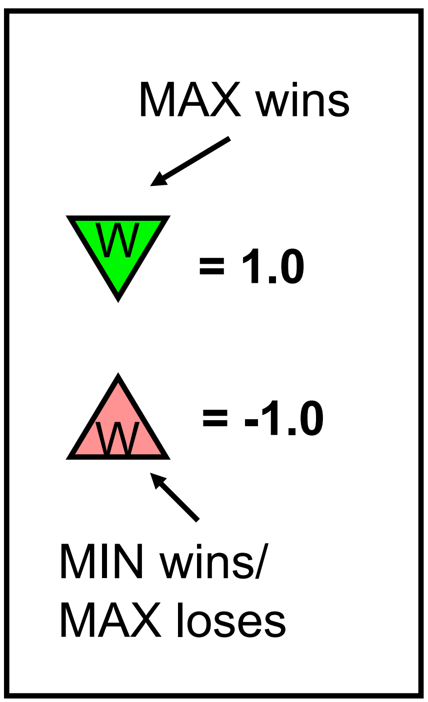

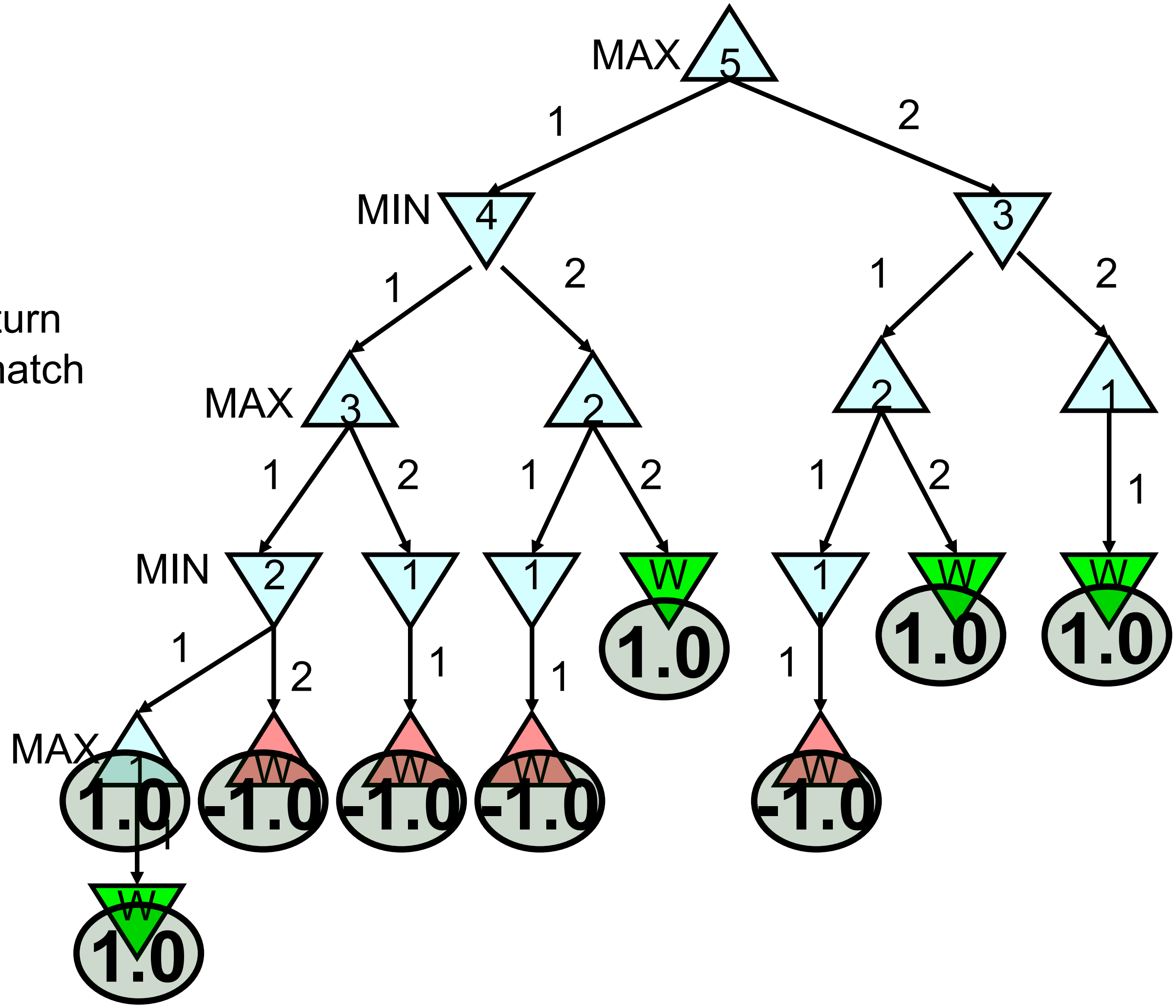

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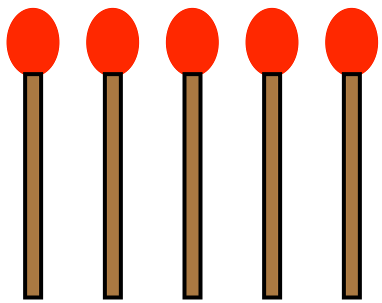

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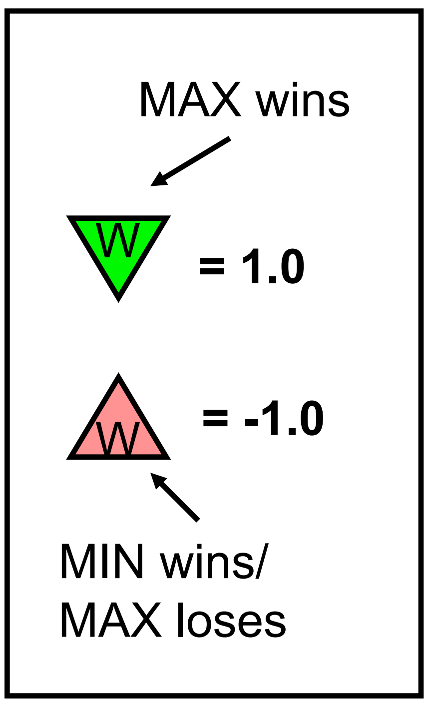

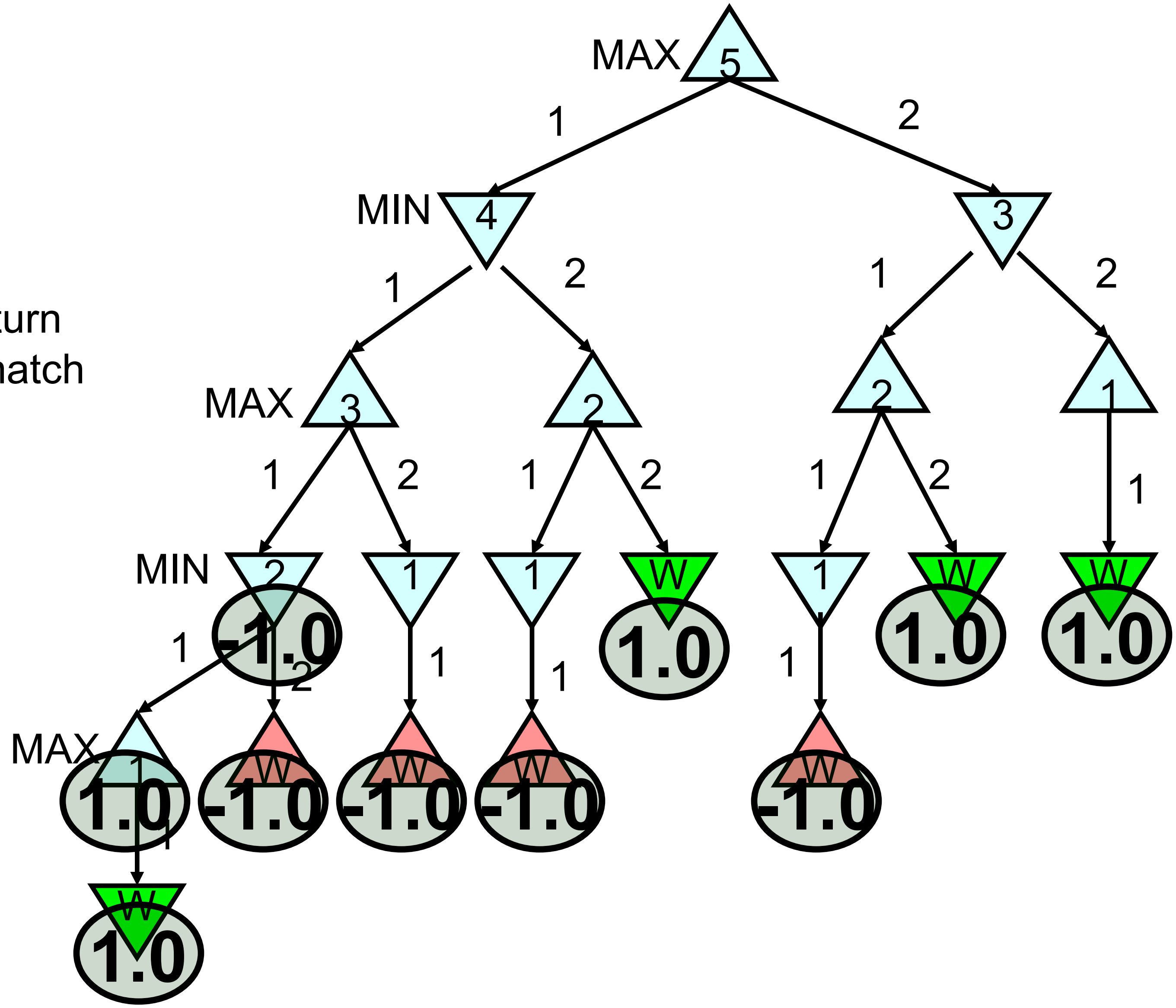

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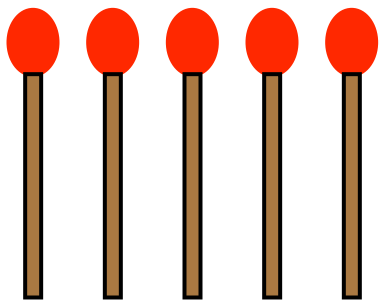

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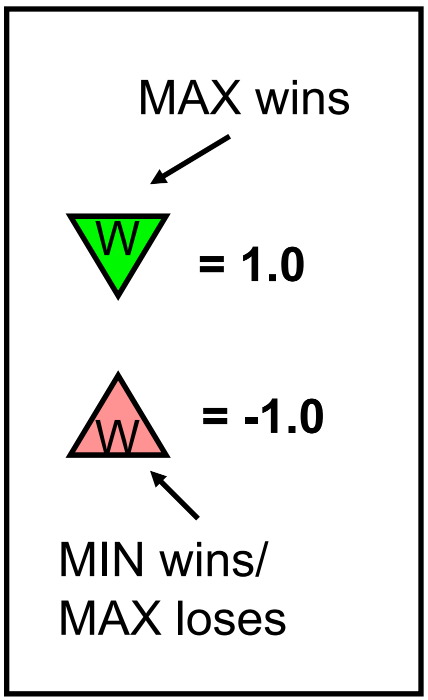

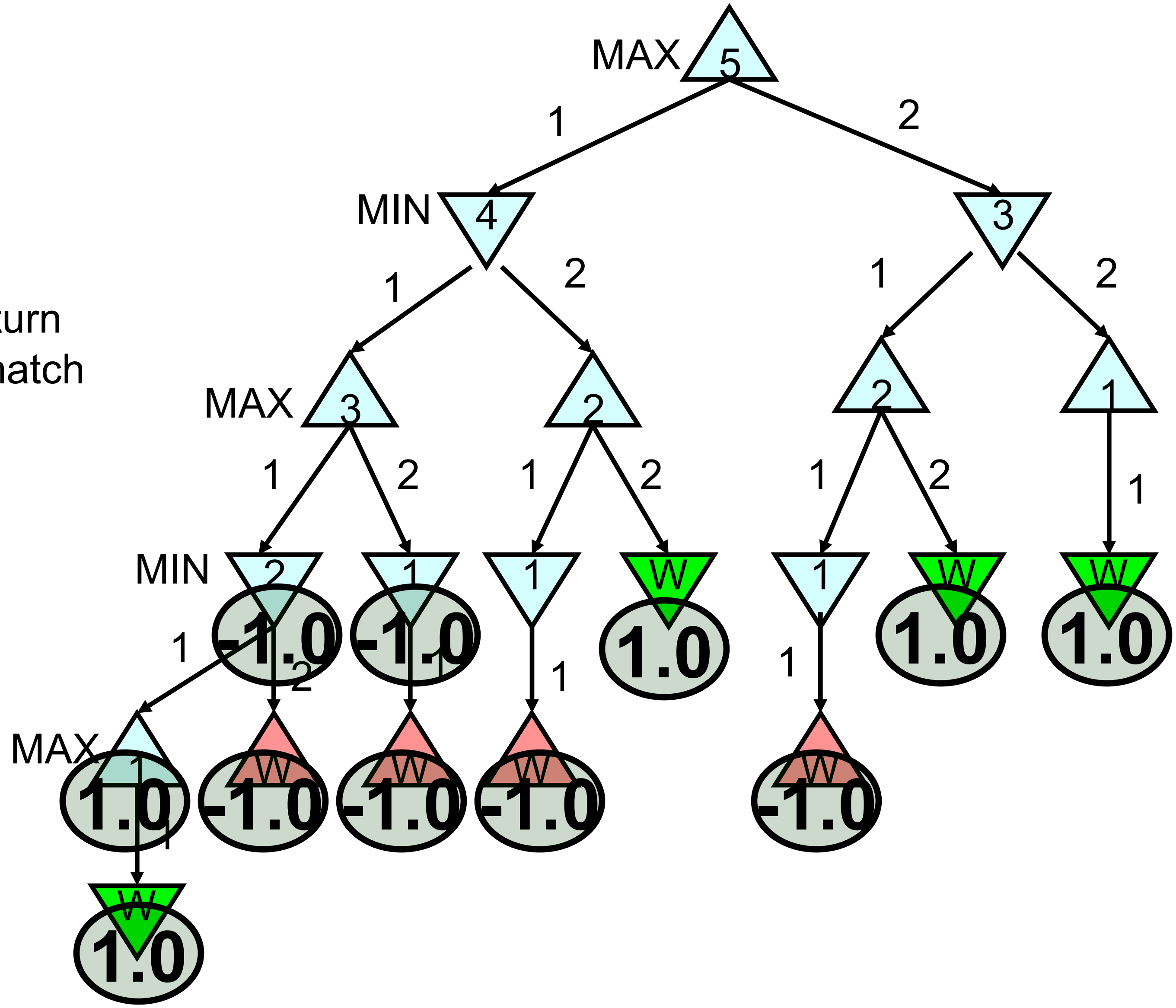

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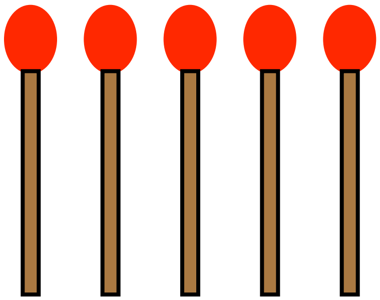

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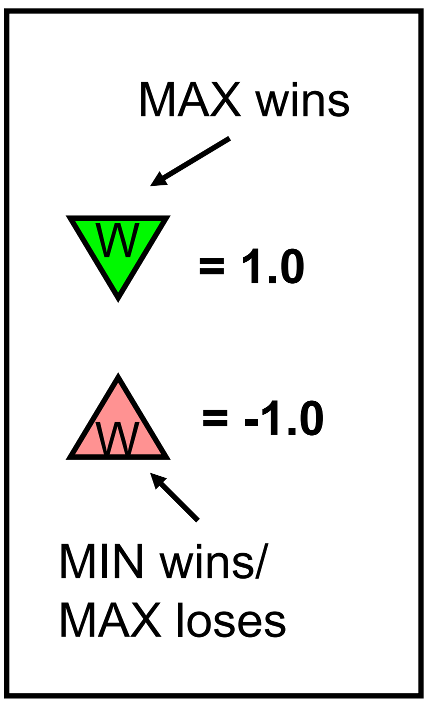

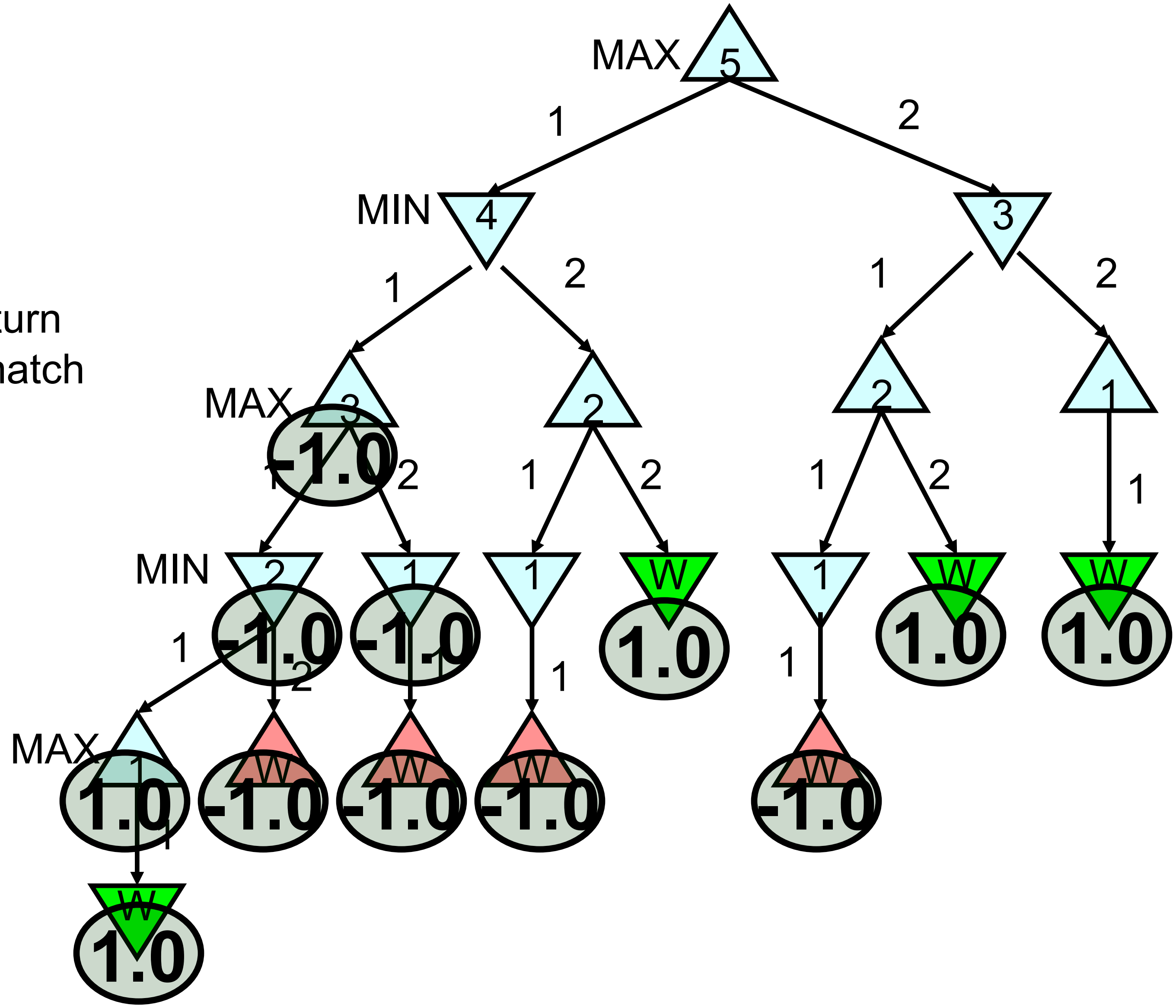

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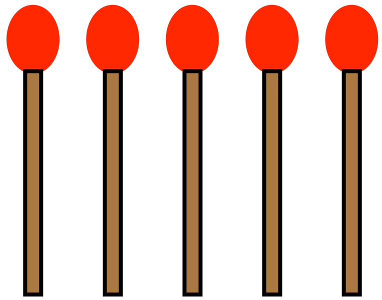

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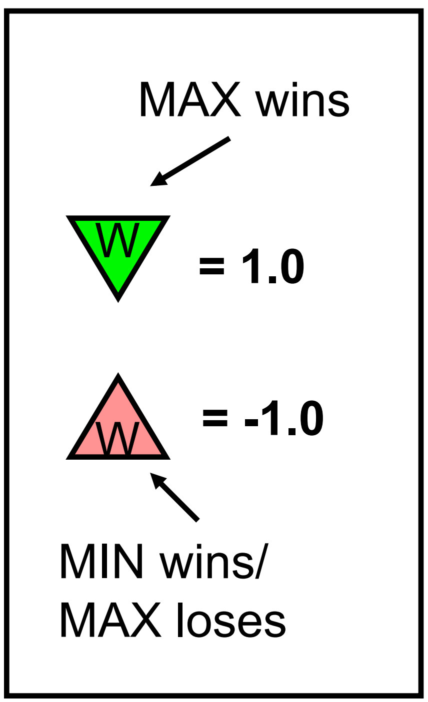

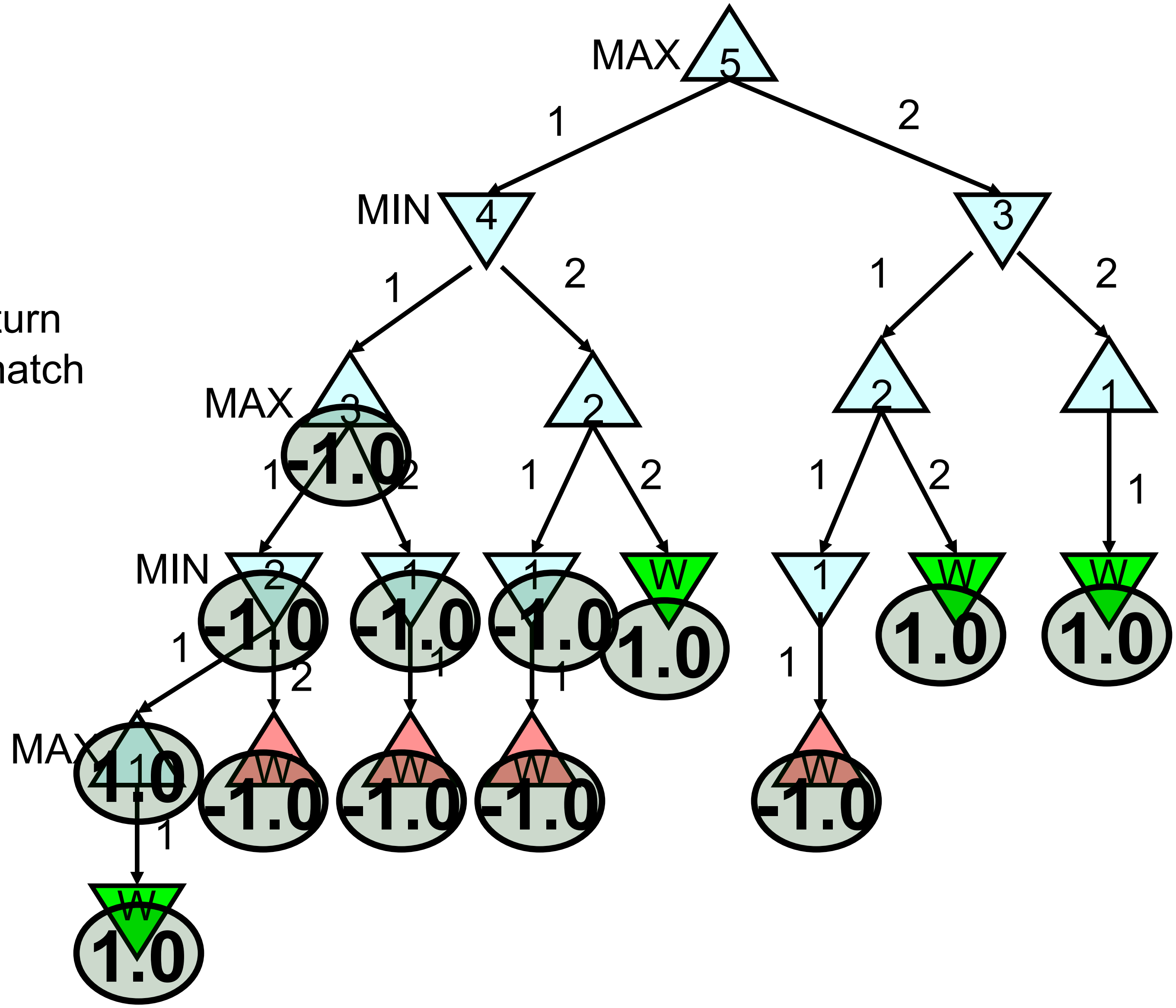

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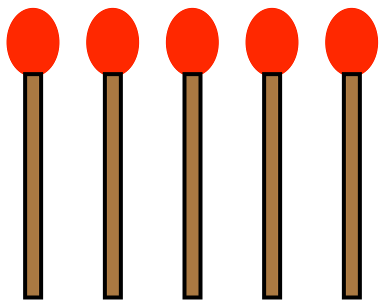

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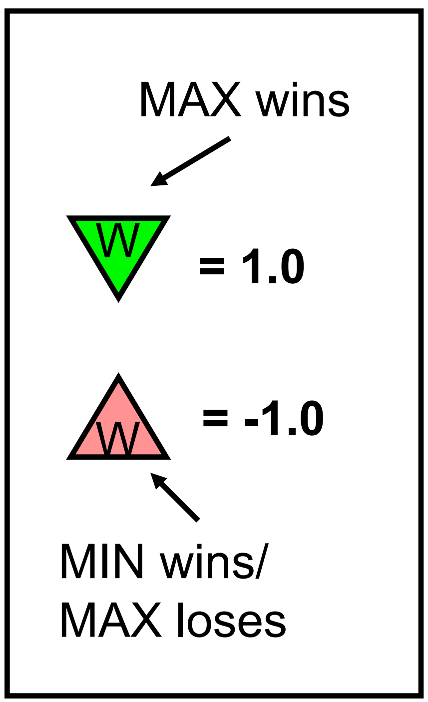

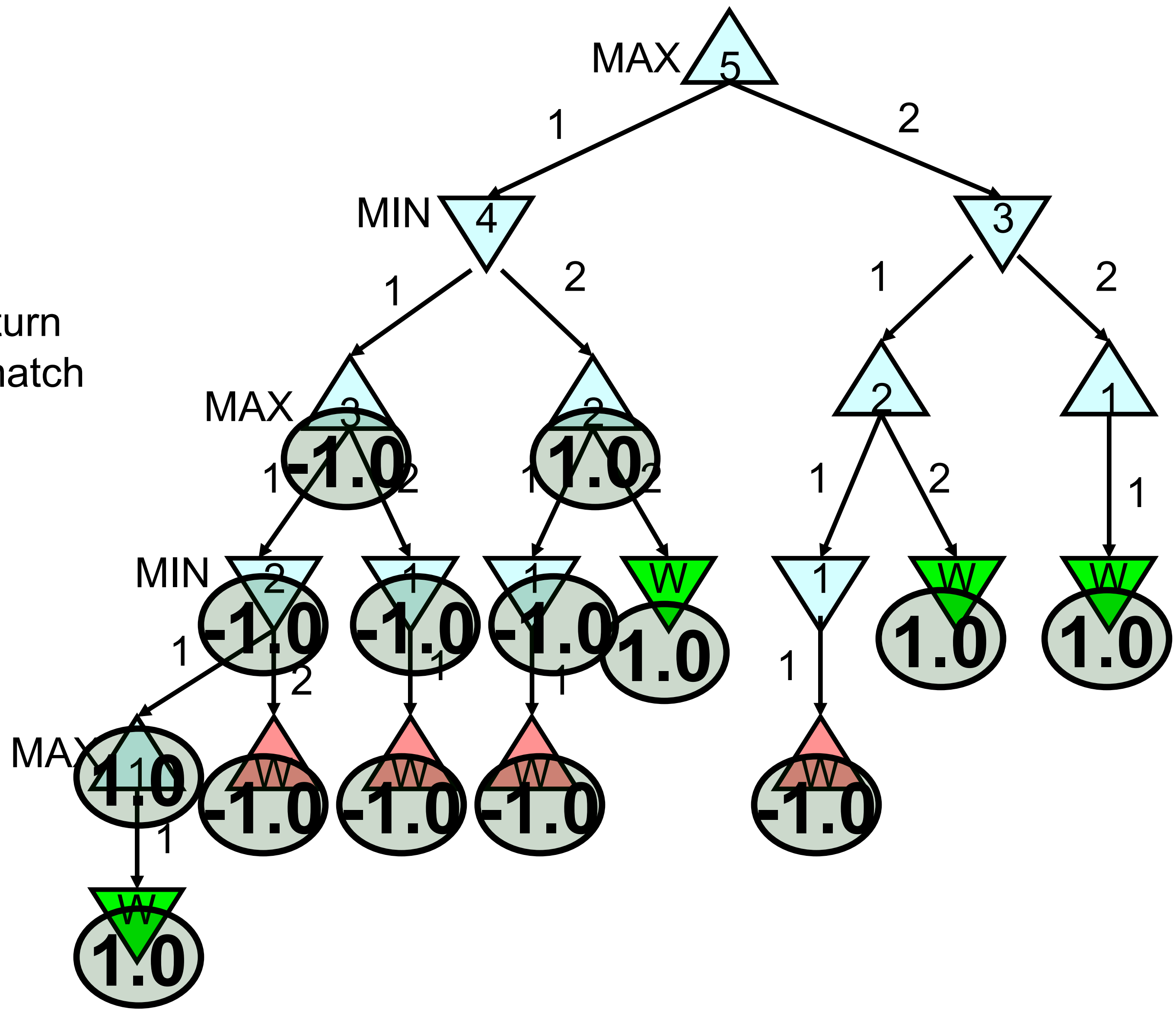

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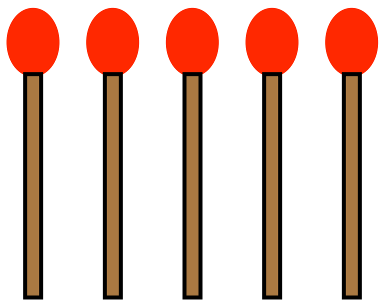

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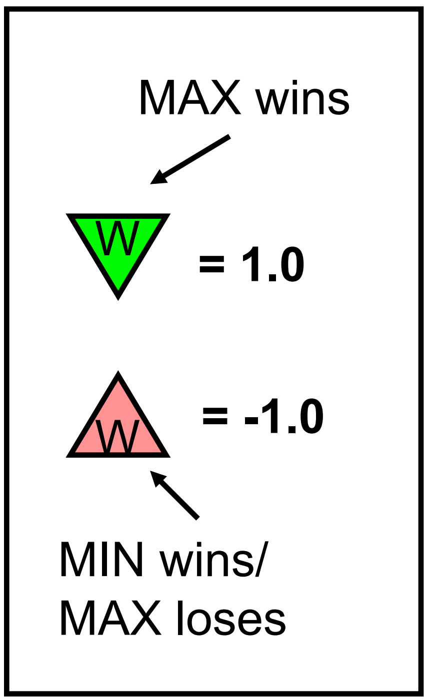

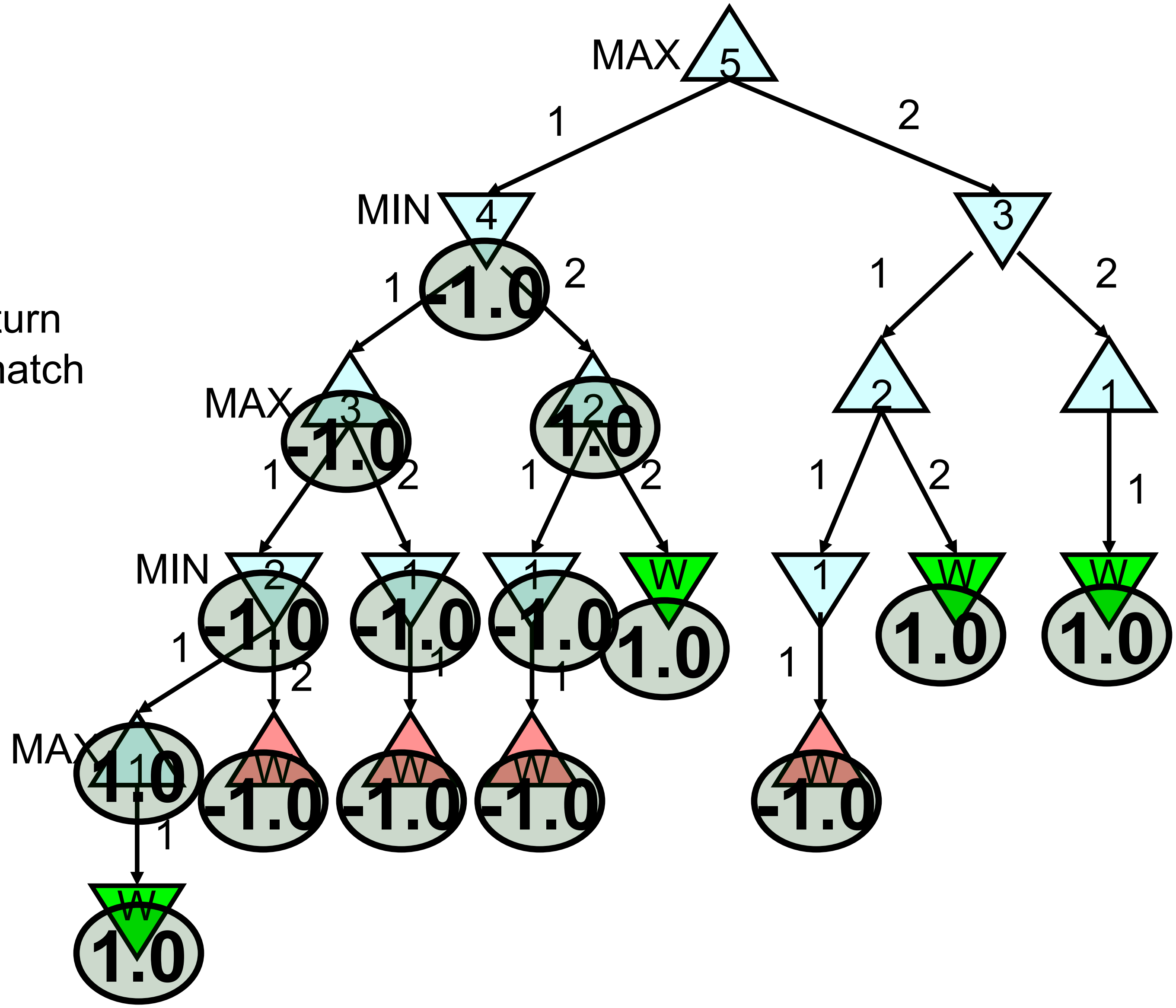

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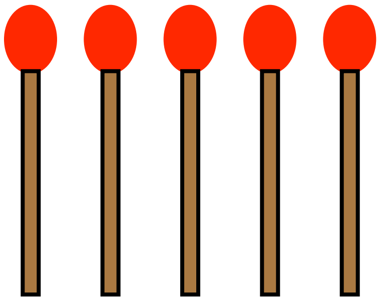

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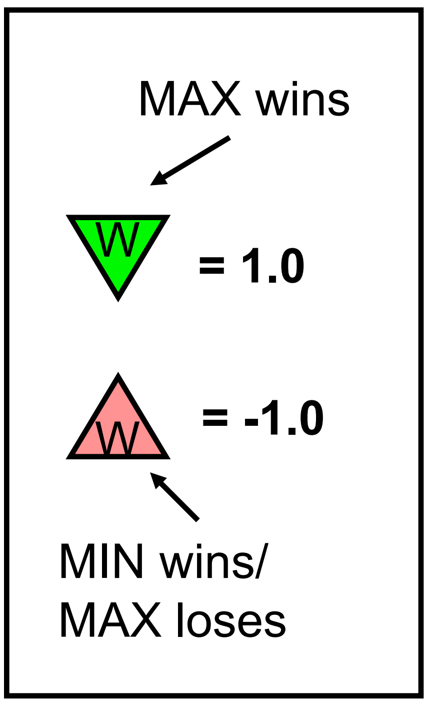

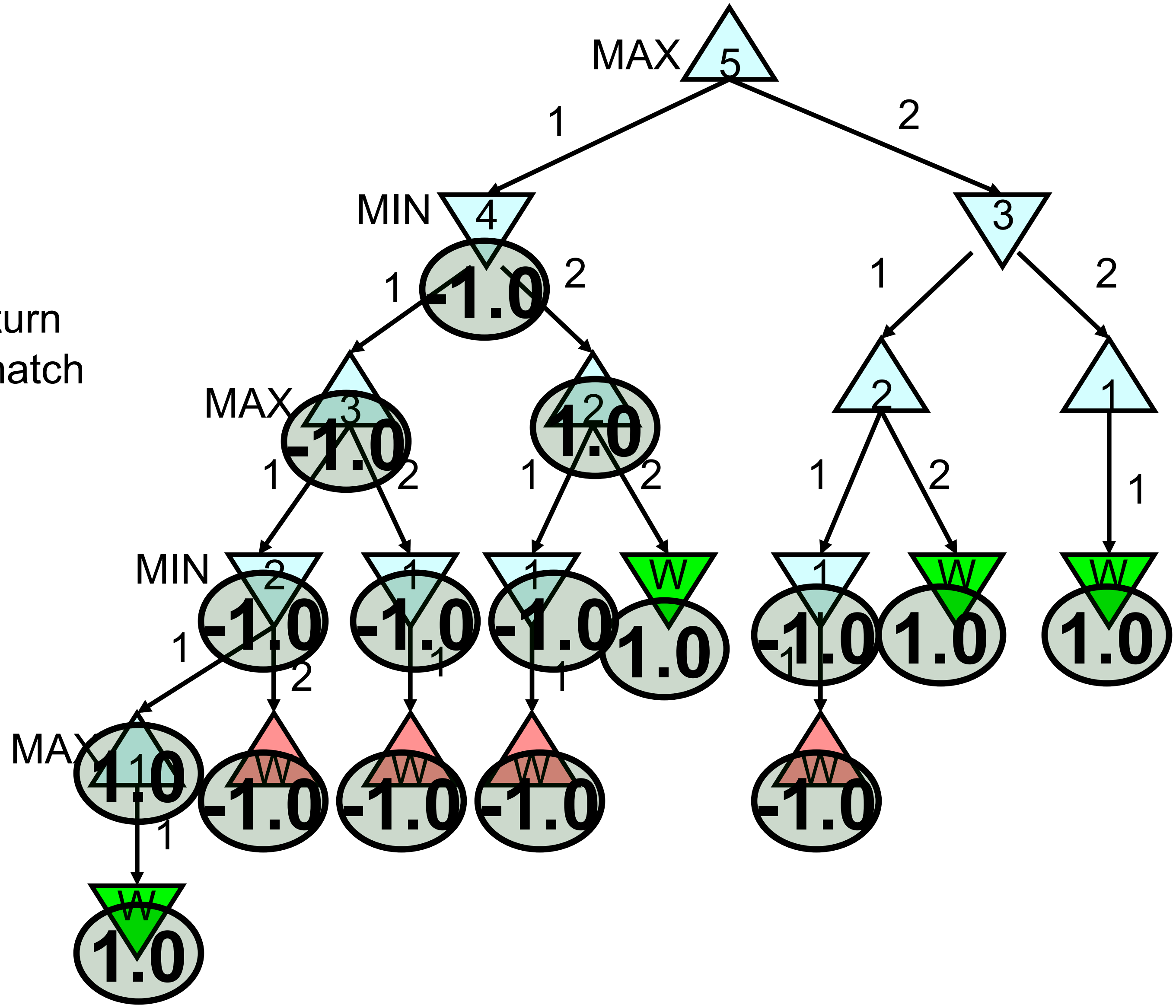

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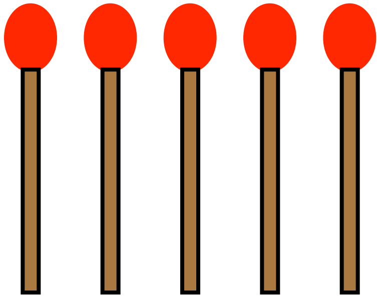

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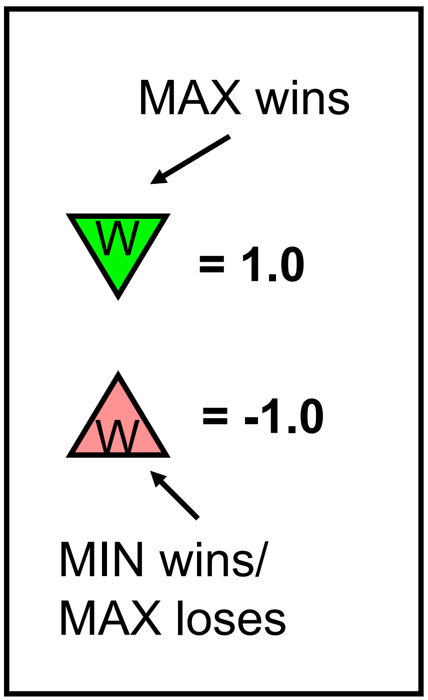

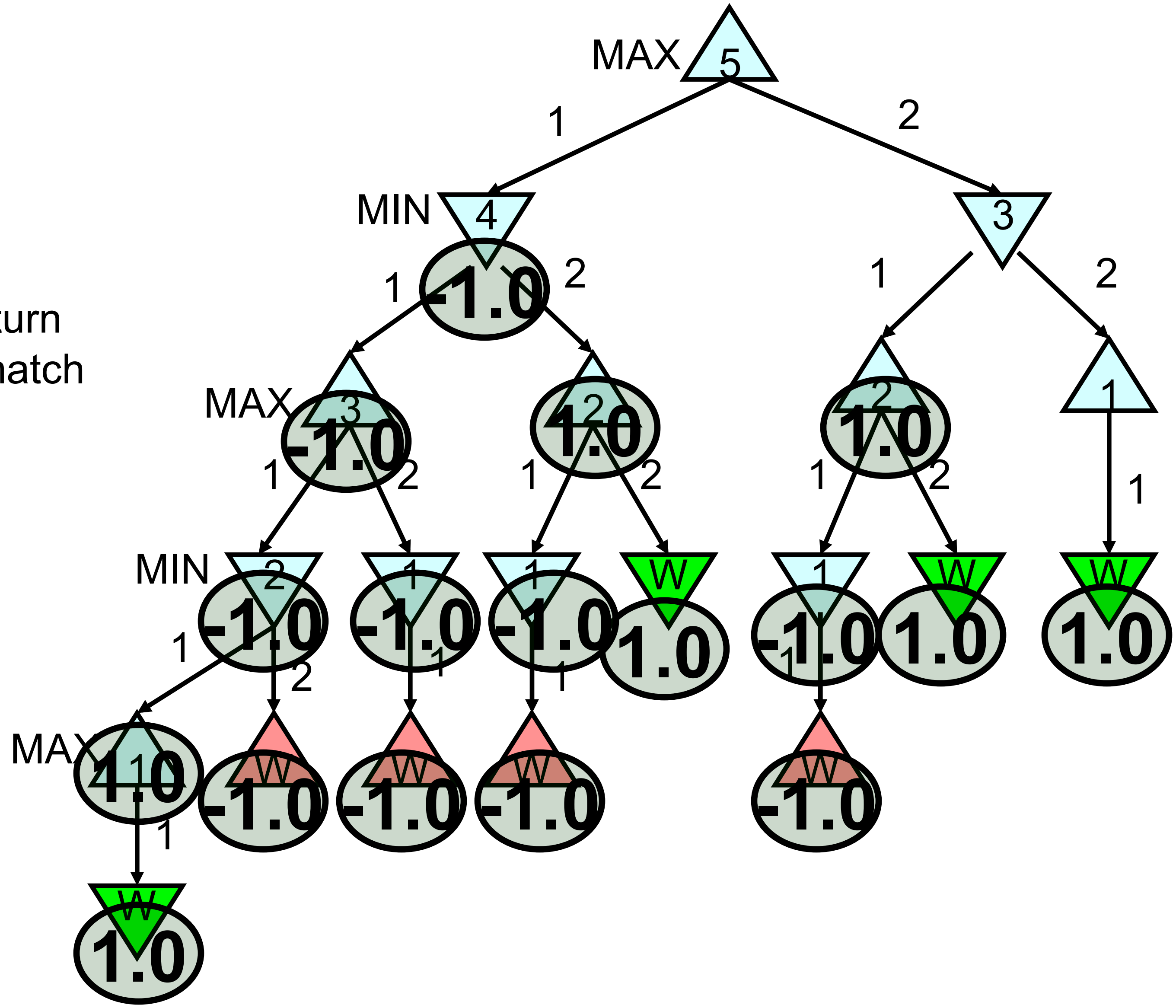

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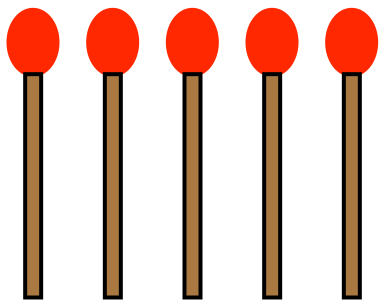

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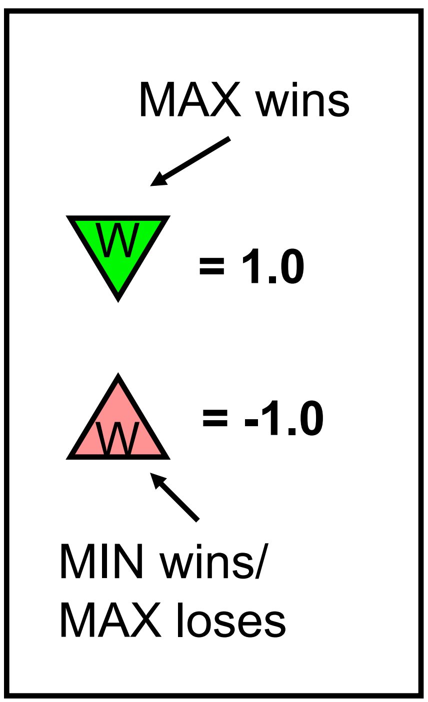

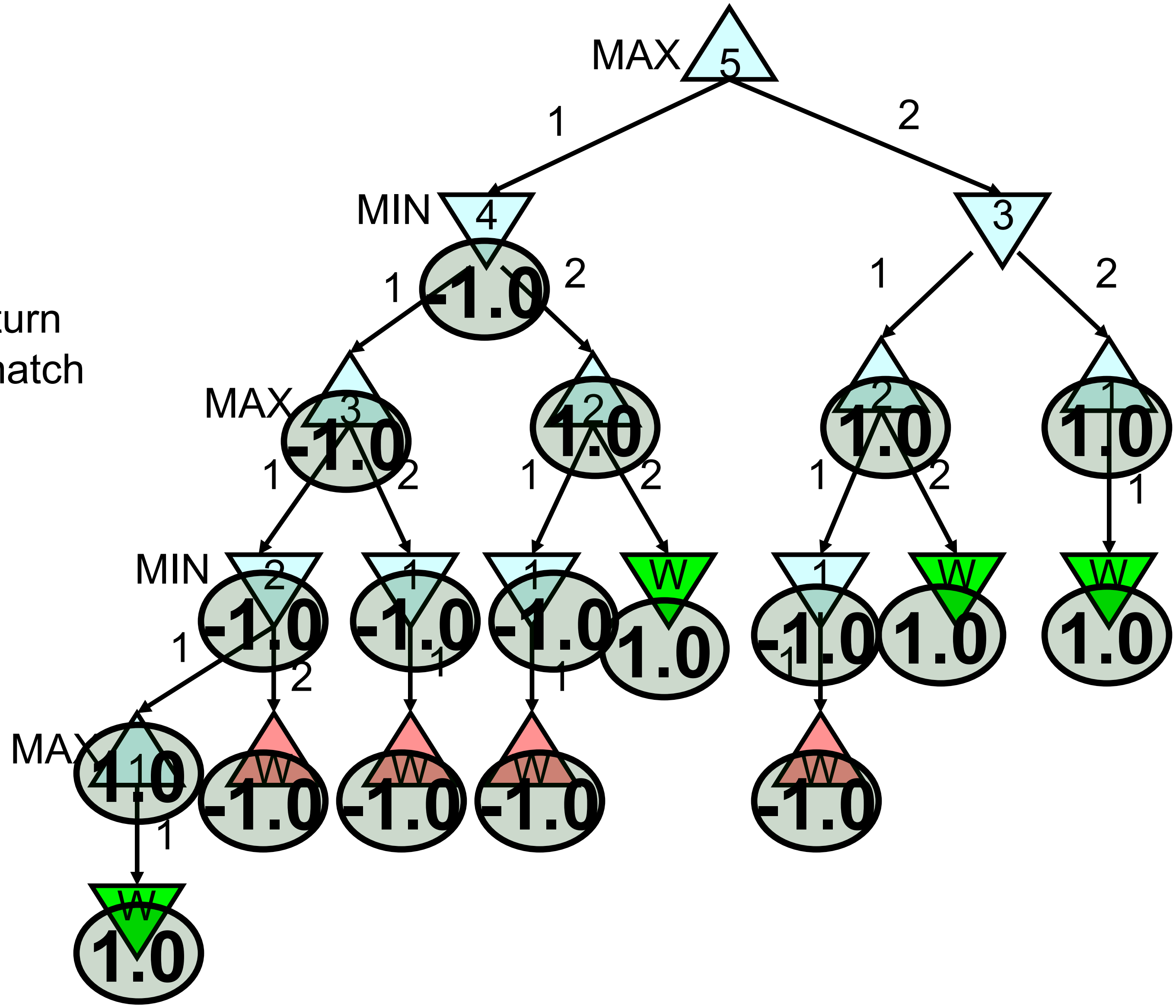

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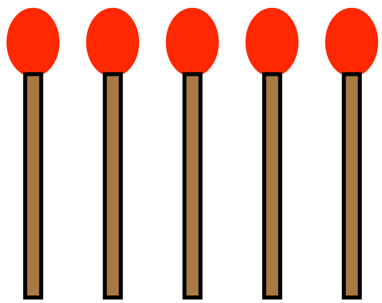

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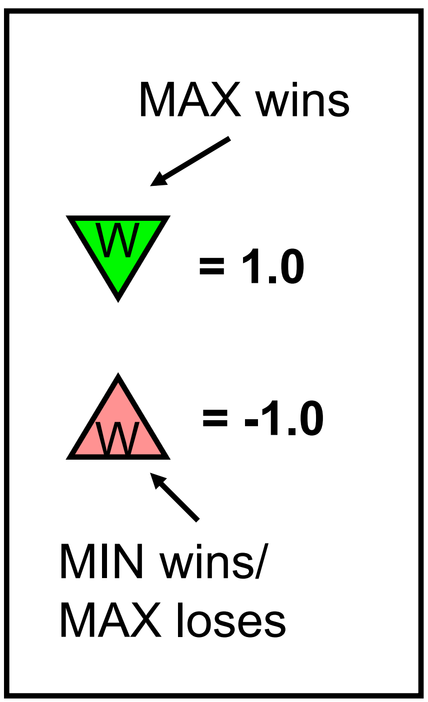
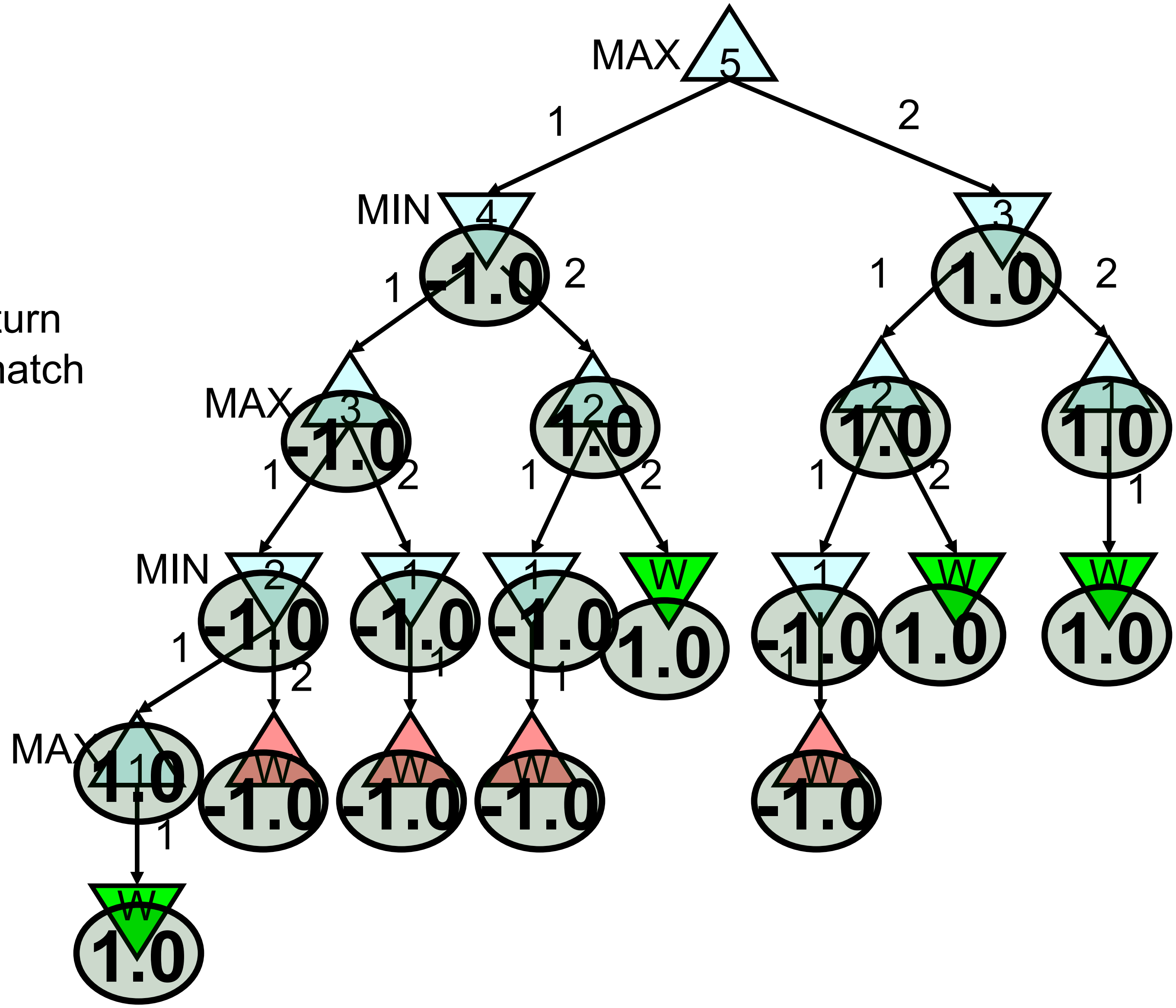
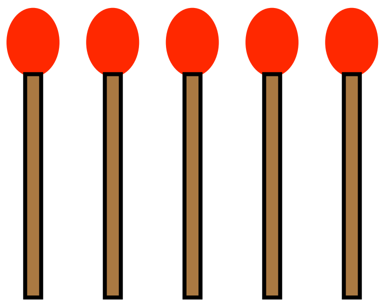


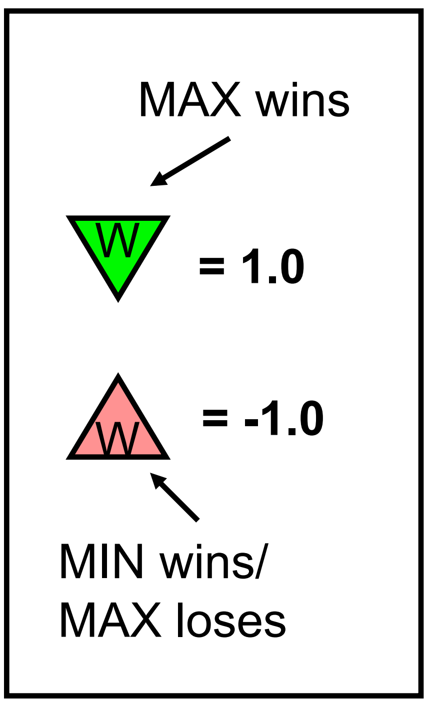


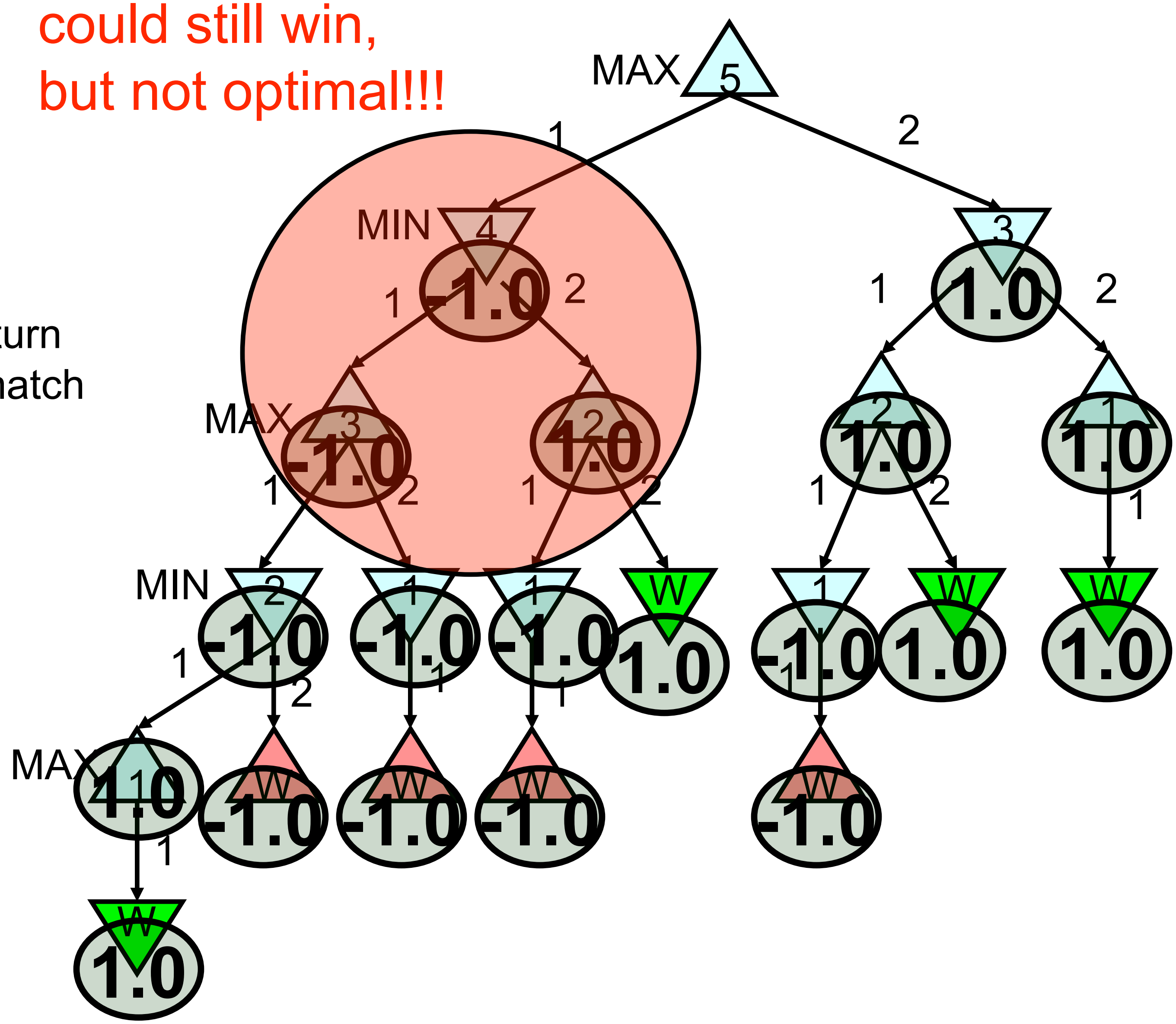
Baby Nim



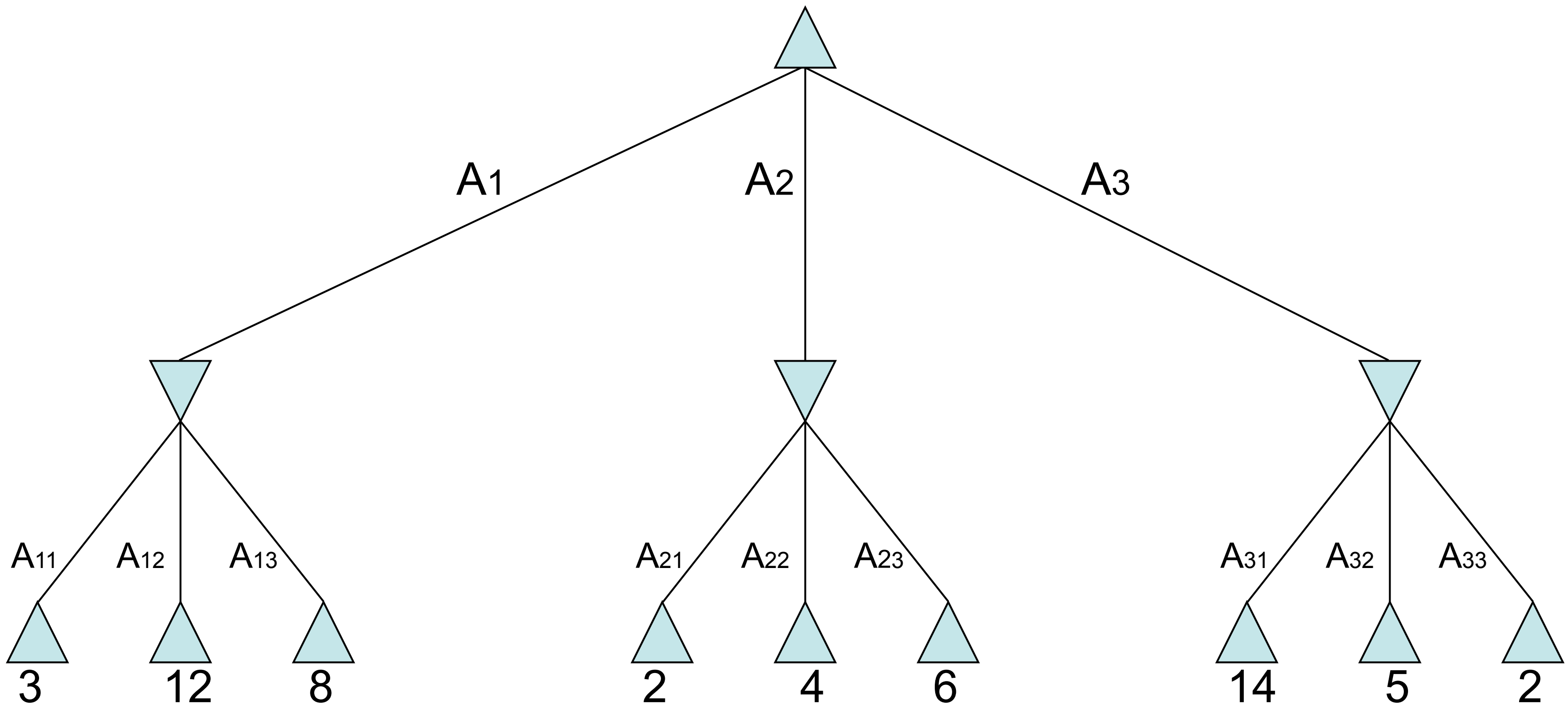
could still win,
but not optimal!!!

Take 1 or 2 at each turn
Goal: take the last match



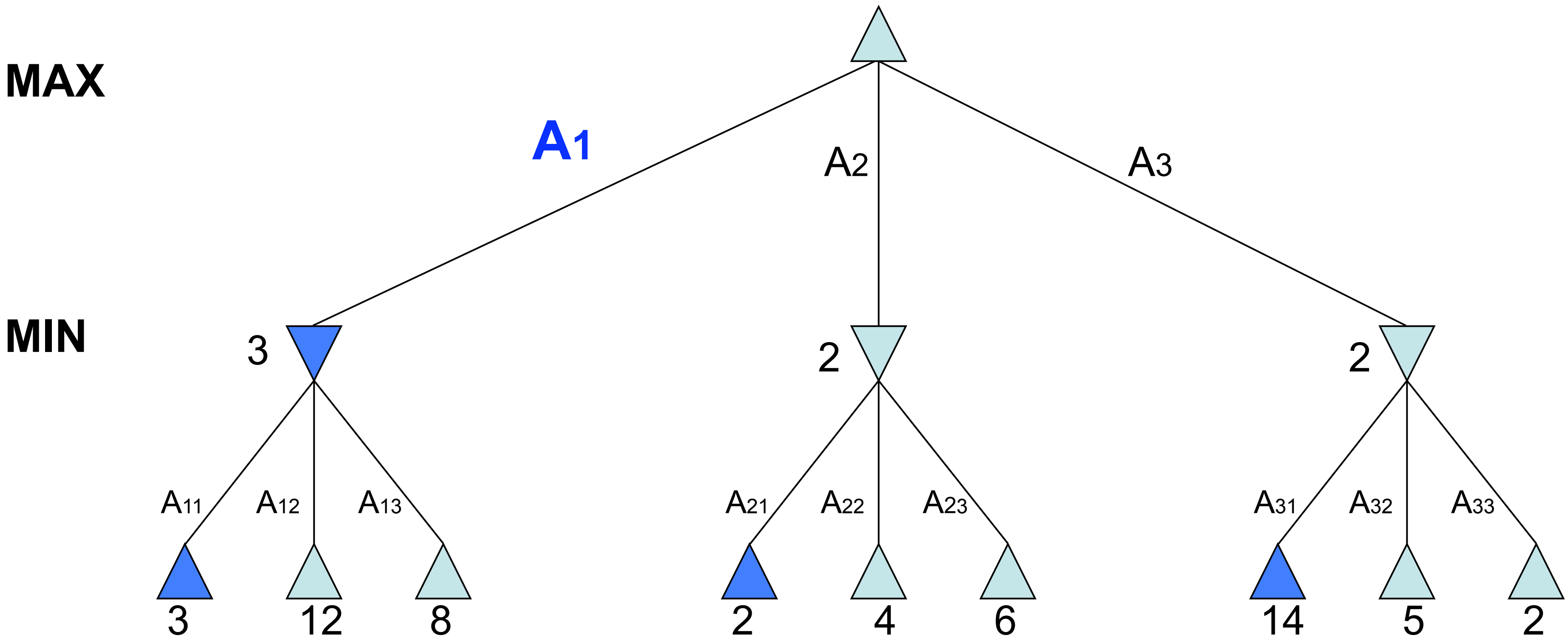


Minimax example 2



Which move should be made: A_1, A_2 or A_3?

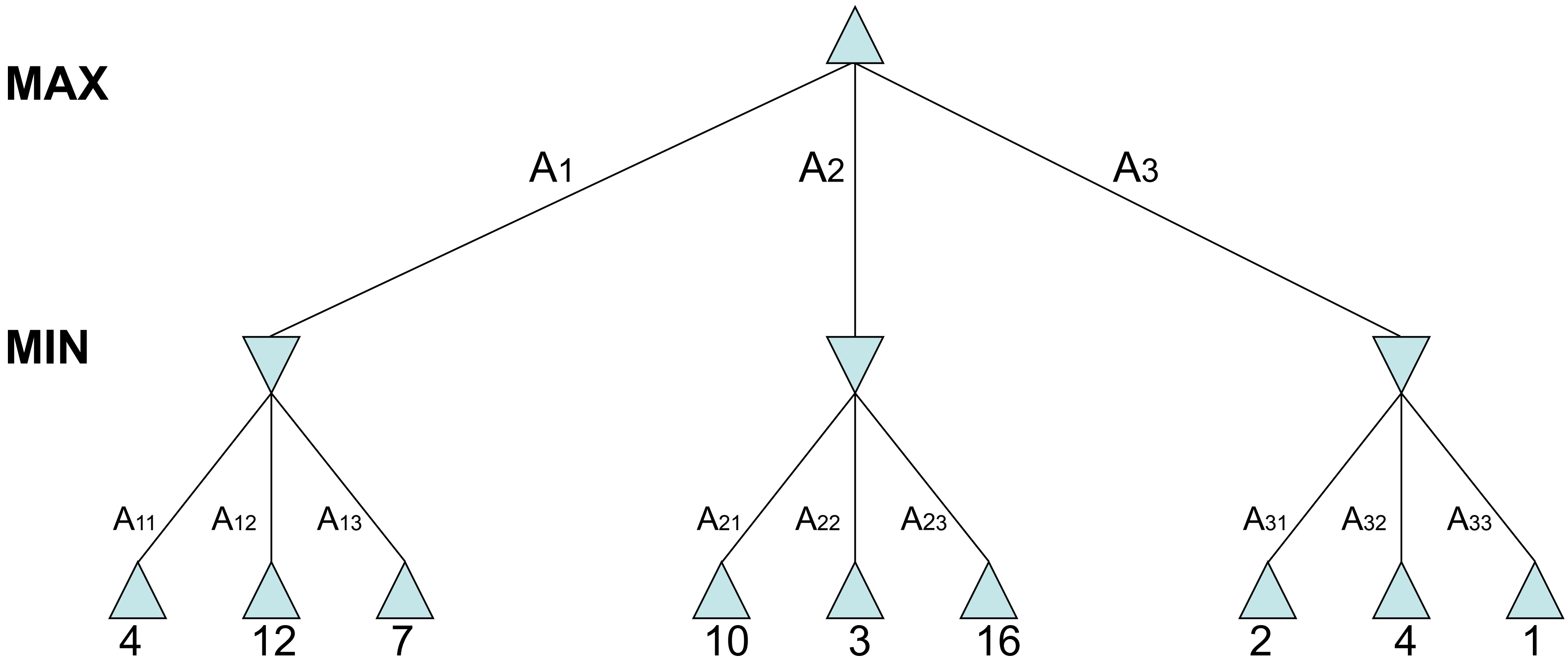
Minimax example 2



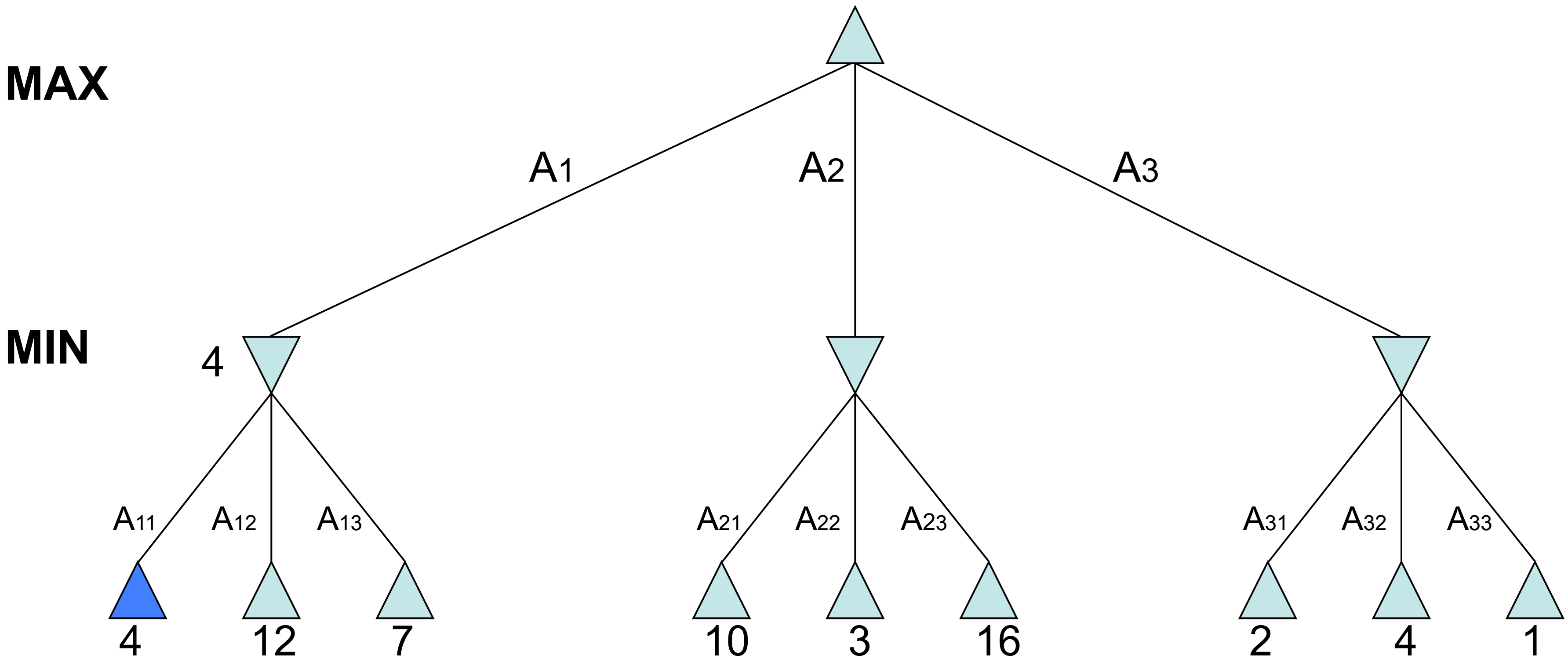
Properties of minimax

- Minimax is optimal!
- Are we done?
  - For chess, $b \approx 35$, $d \approx 100$ for reasonable games → exact solution completely infeasible
  - Is minimax feasible for Mancala or Tic Tac Toe?
    - Mancala: 6 possible moves. average depth of 40, so 6^{40} which is on the edge
    - Tic Tac Toe: branching factor of 4 (on average) and depth of 9… yes!
- Ideas?
  - pruning!
  - improved state utility/evaluation functions

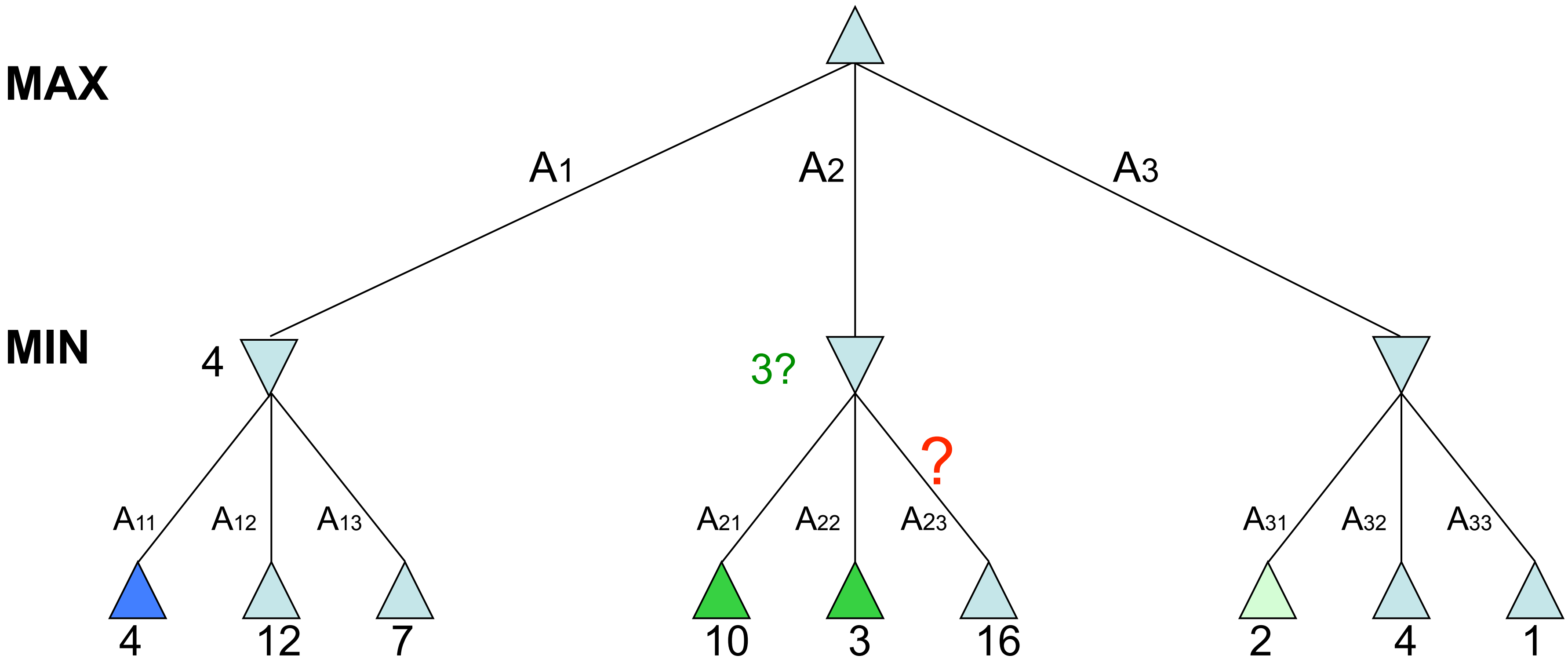
Pruning: do we have to traverse the whole tree?



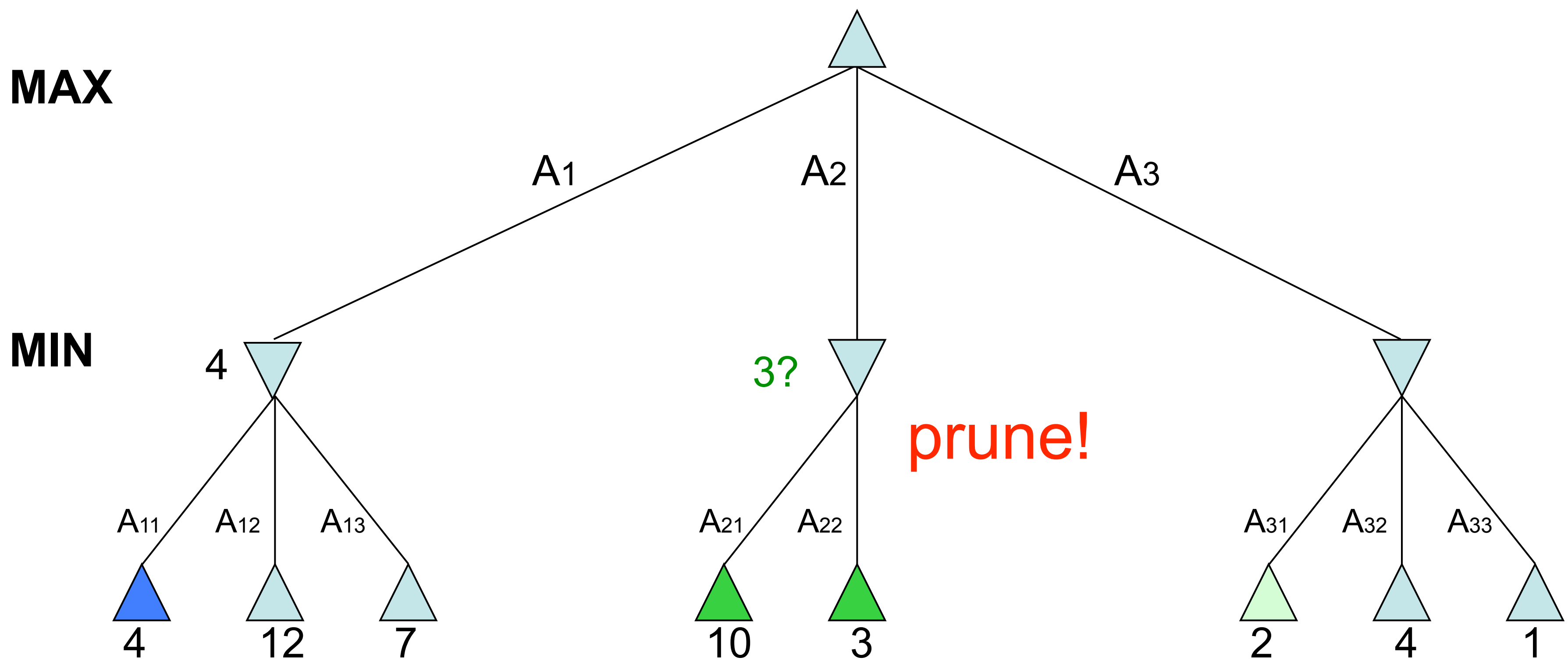
Pruning: do we have to traverse the whole tree?



Minimax example 2

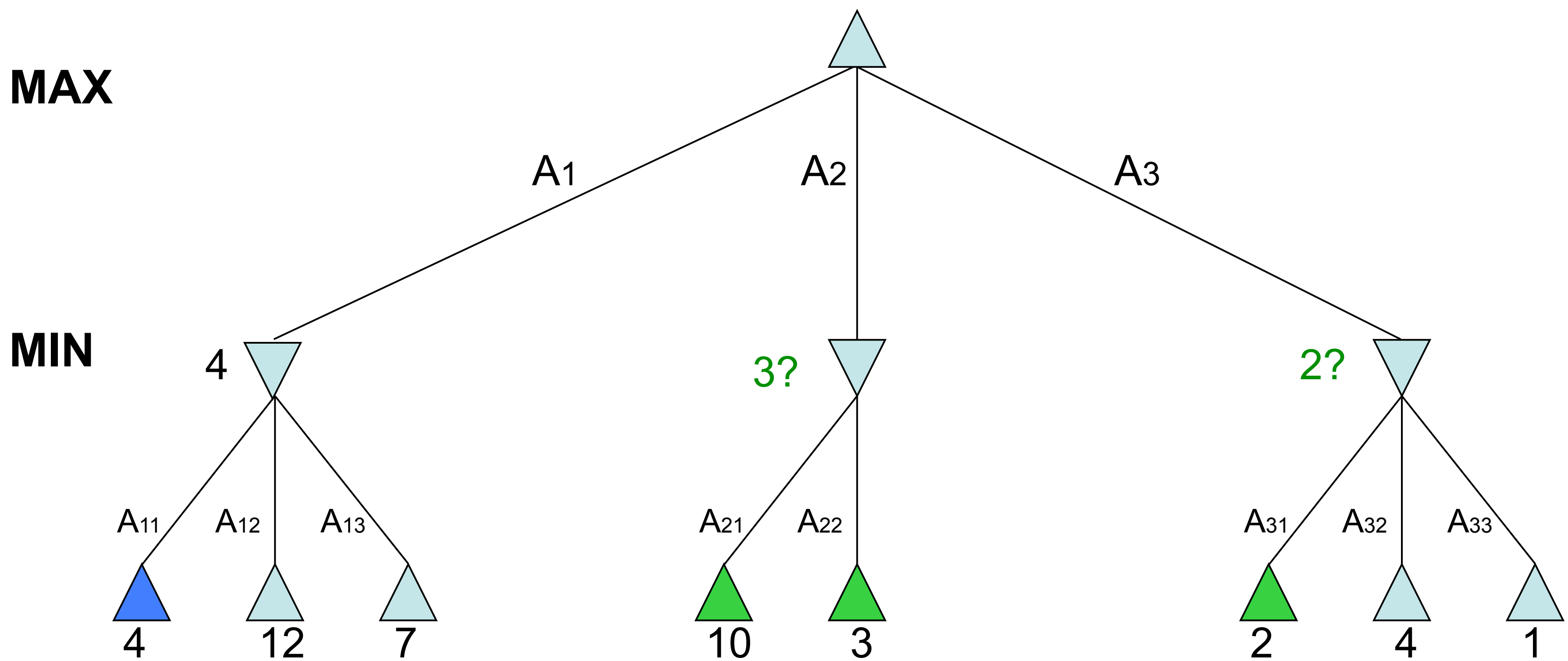


Minimax example 2

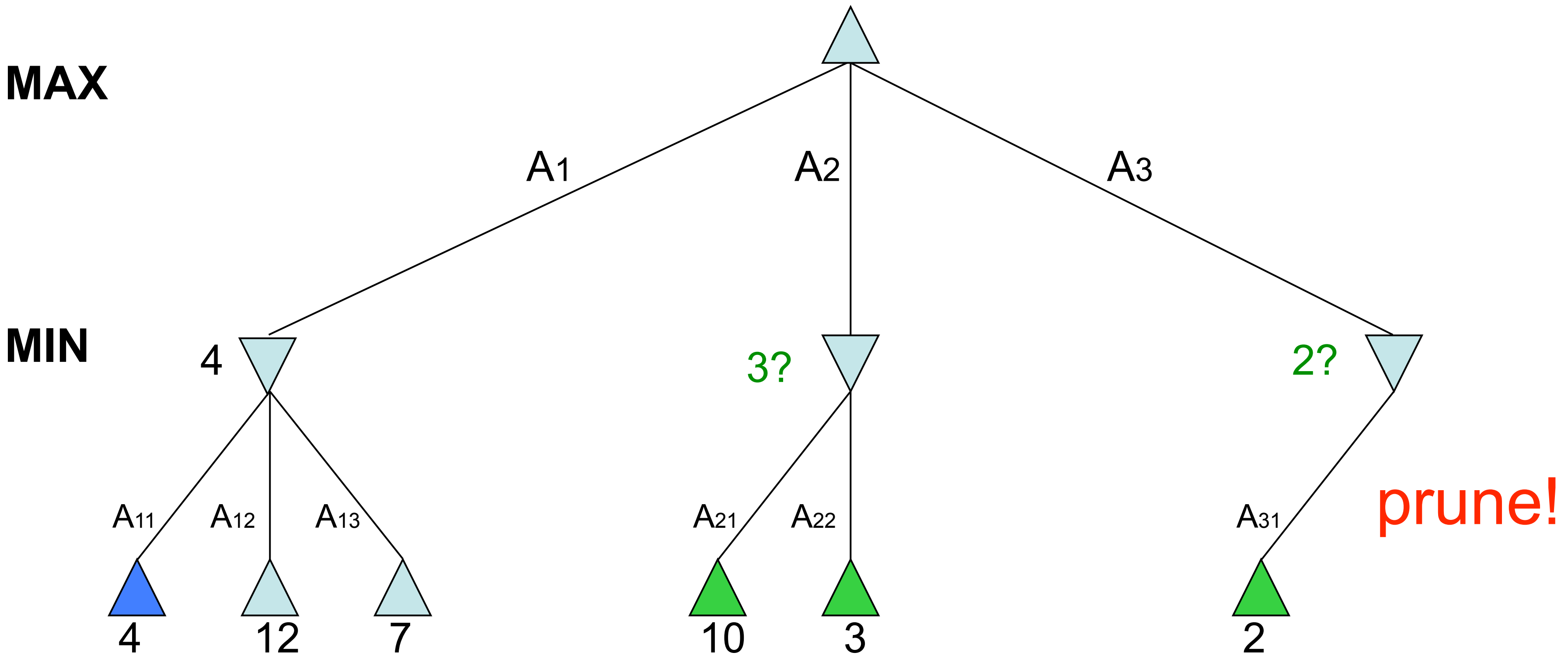


Any others if we continue?

Minimax example 2



Minimax example 2

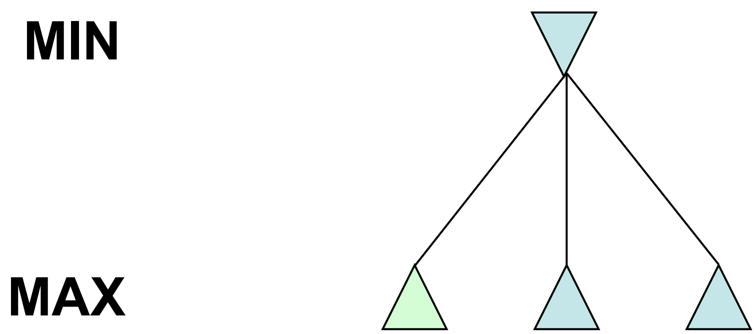


Alpha-Beta pruning

- An optimal pruning strategy
  - only prunes paths that are suboptimal (i.e. wouldn’t be chosen by an optimal playing player)
  - returns the *same* result as minimax, but faster
- As we go, keep track of the best and worse along a path
  - alpha = best choice we’ve found so far for MAX
  - beta = best choice we’ve found so far for MIN

Alpha-Beta pruning

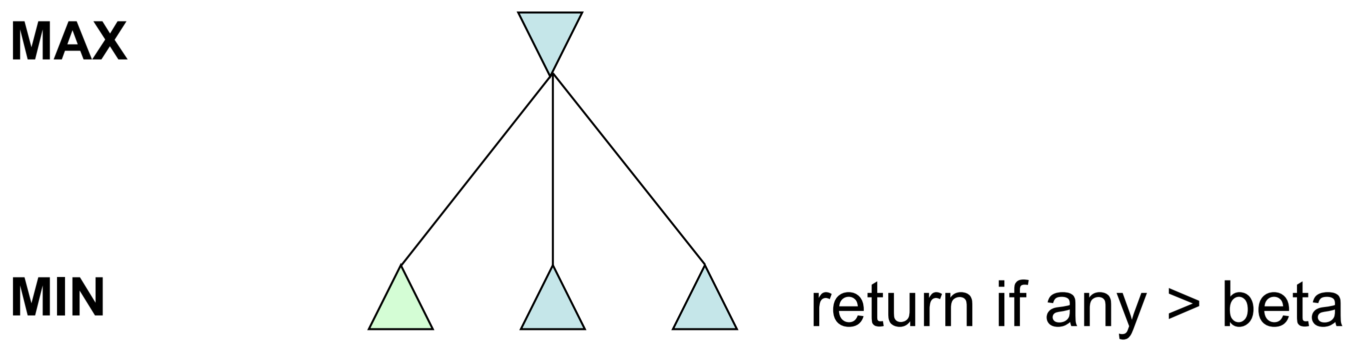
- alpha = best choice we've found so far for MAX
- Using alpha and beta to prune:
  - We're examining MIN's options for a ply. To do this, we're examining all possible moves for MAX. If we find a value for one of MAX's moves that is **less than alpha**, return. (MIN could do better down this path)



return if any < alpha

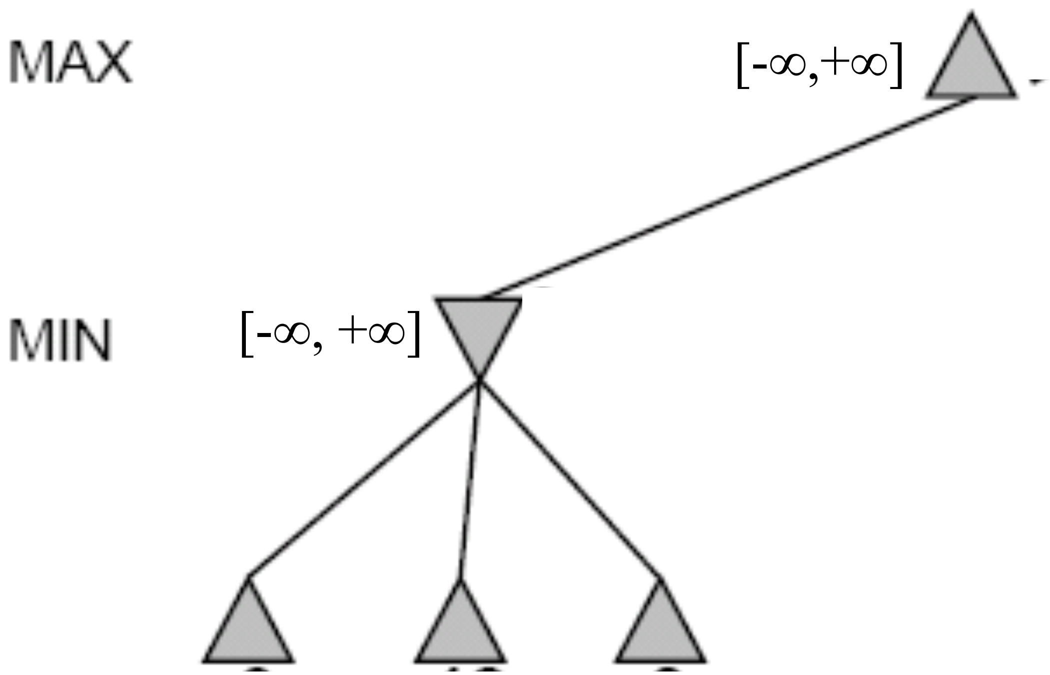
Alpha-Beta pruning

- beta = best choice we've found so far for MIN
- Using alpha and beta to prune:
  - We're examining MAX's options for a ply. To do this, we're examining all possible moves for MIN. If we find a value for one of MIN's possible moves that is **greater than beta**, return. (MIN won't end up down here)



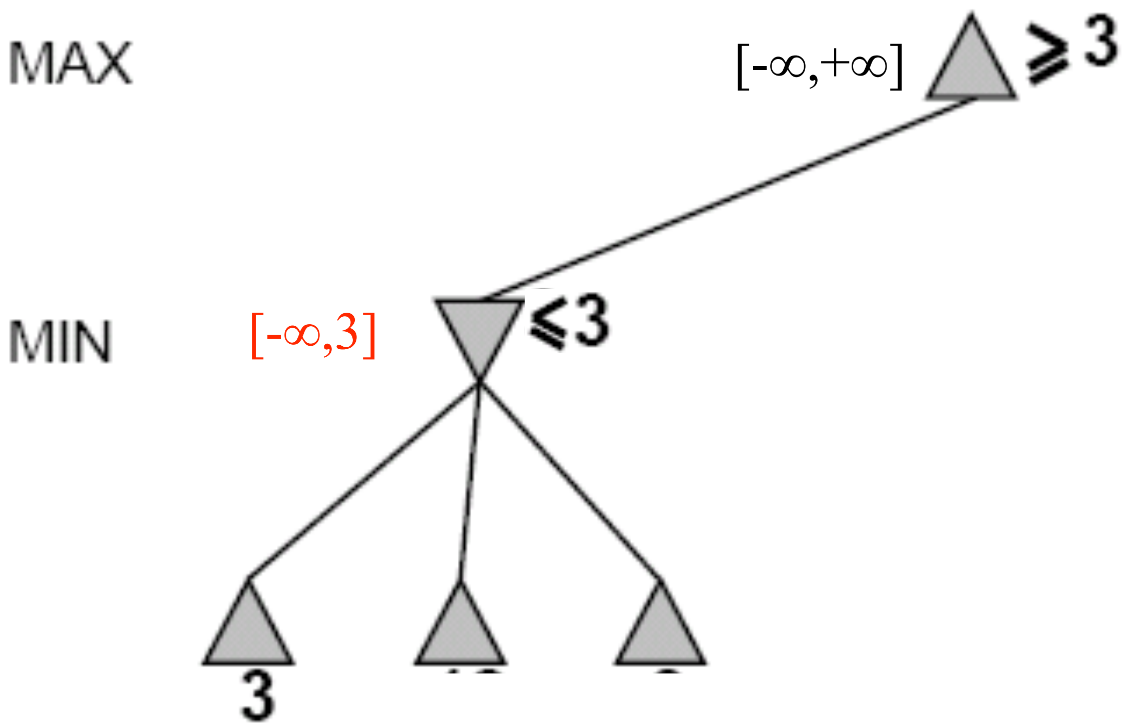
Alpha-Beta pruning

Do DF-search until first leaf



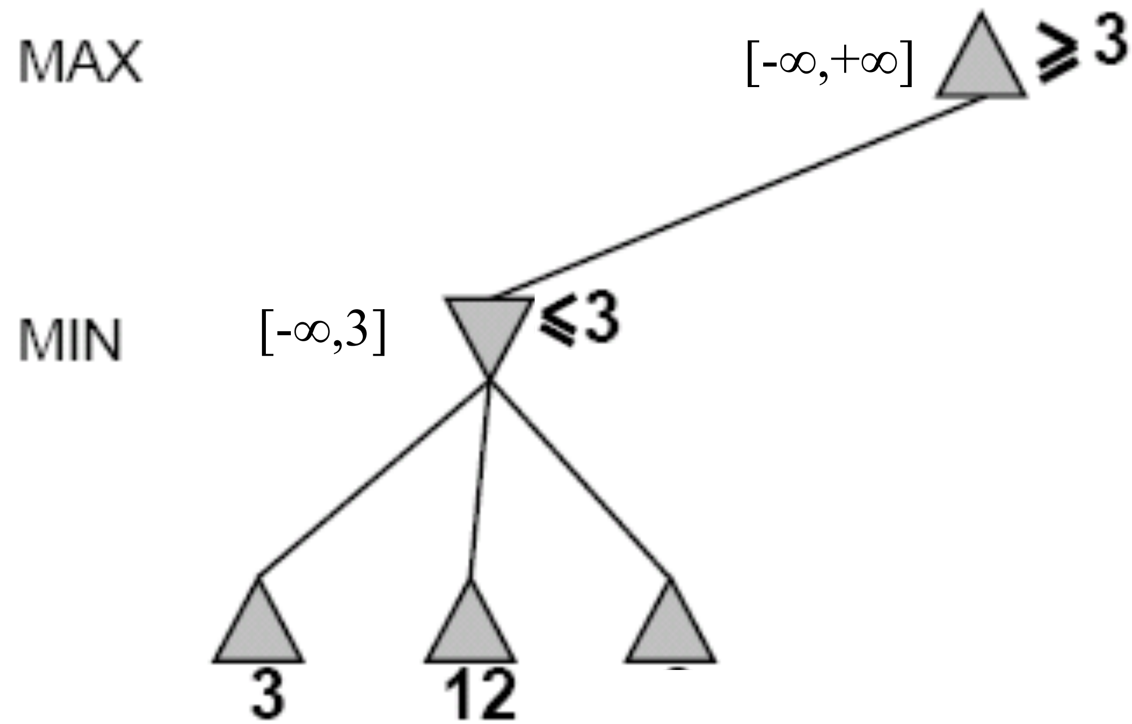
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



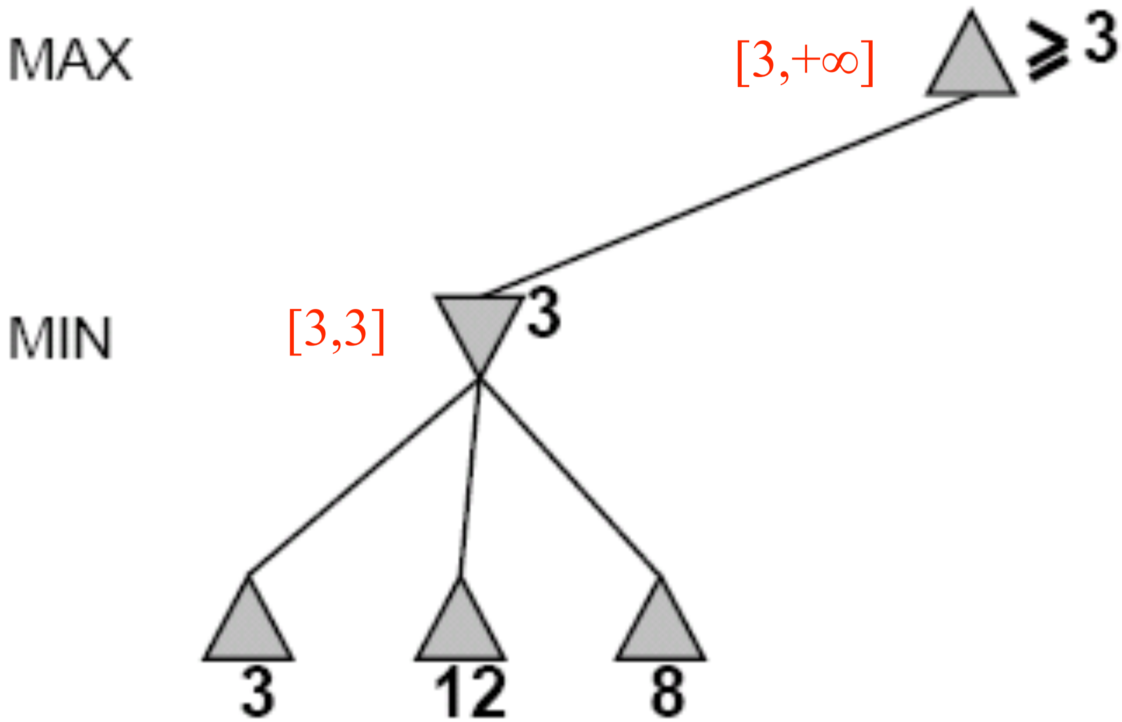
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



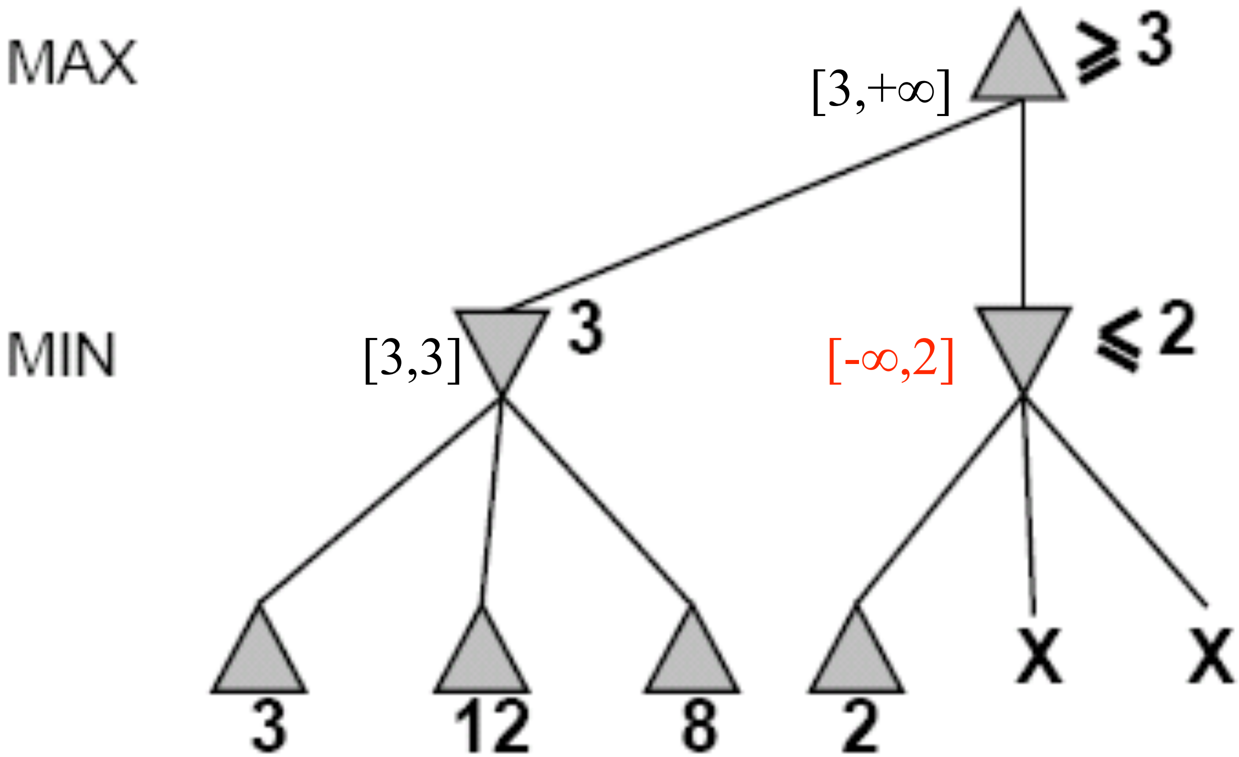
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



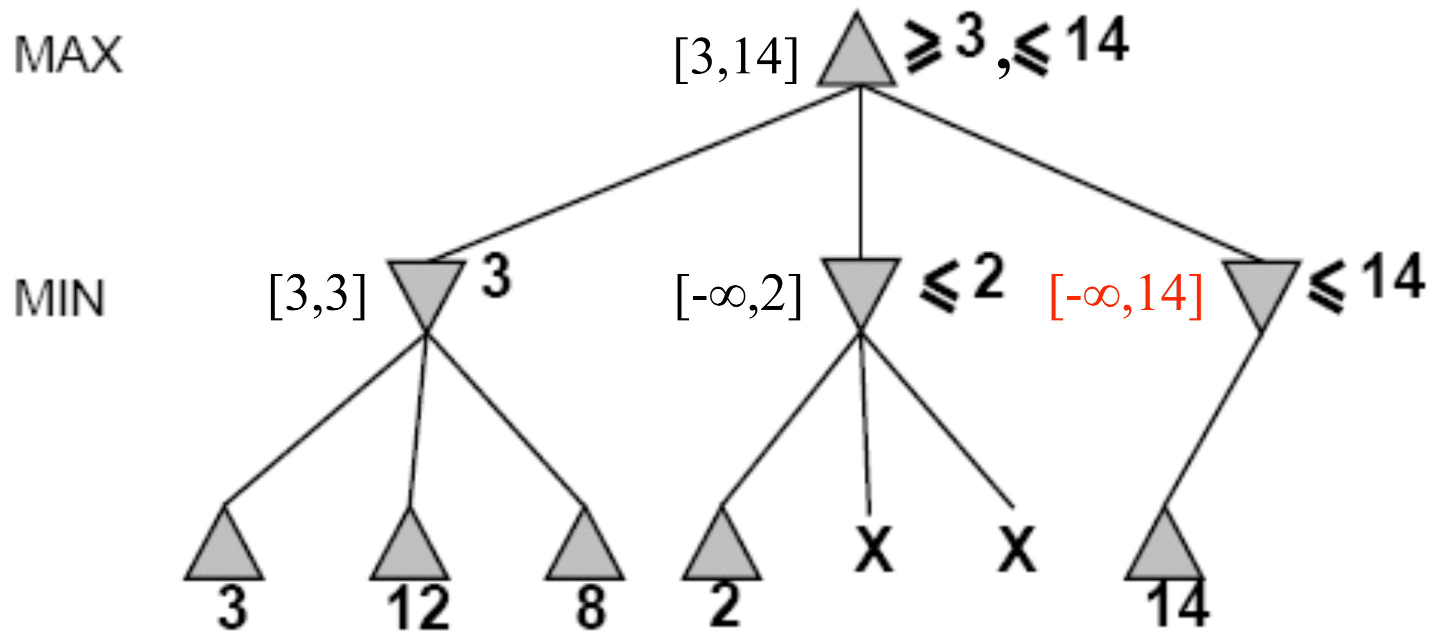
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



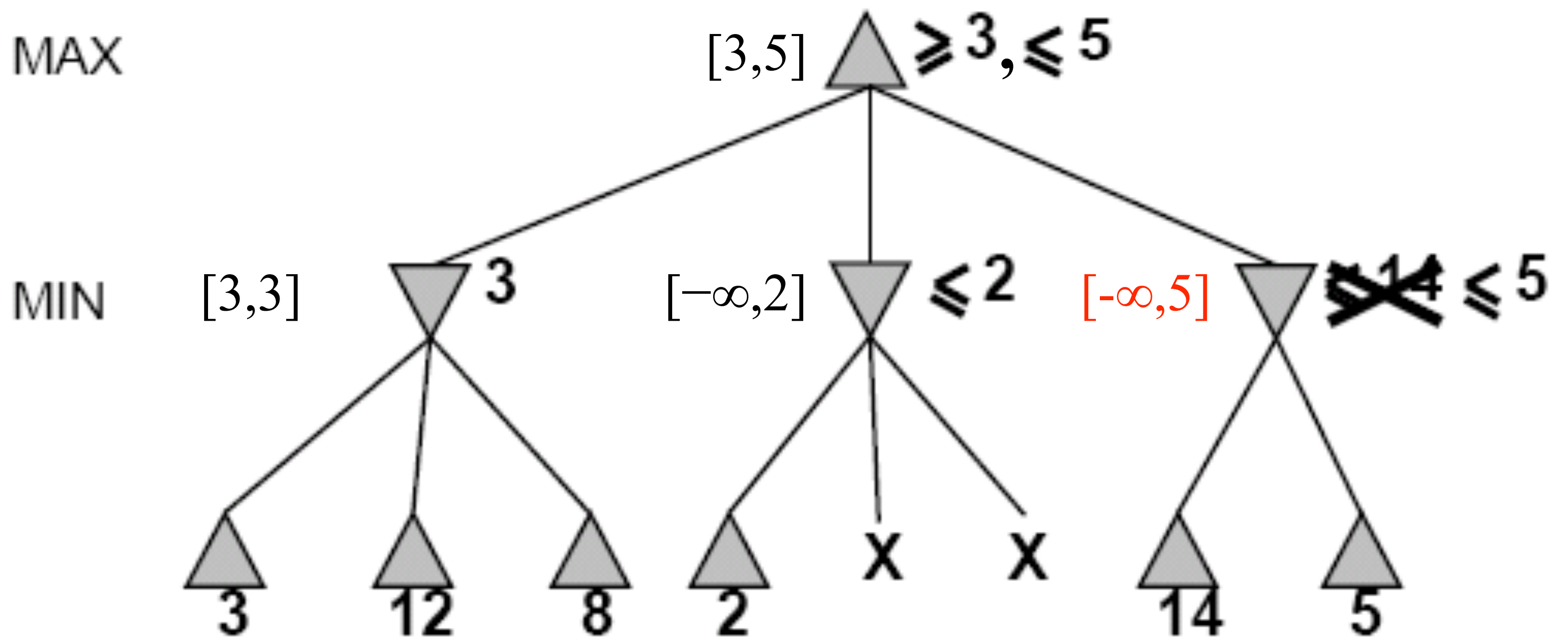
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



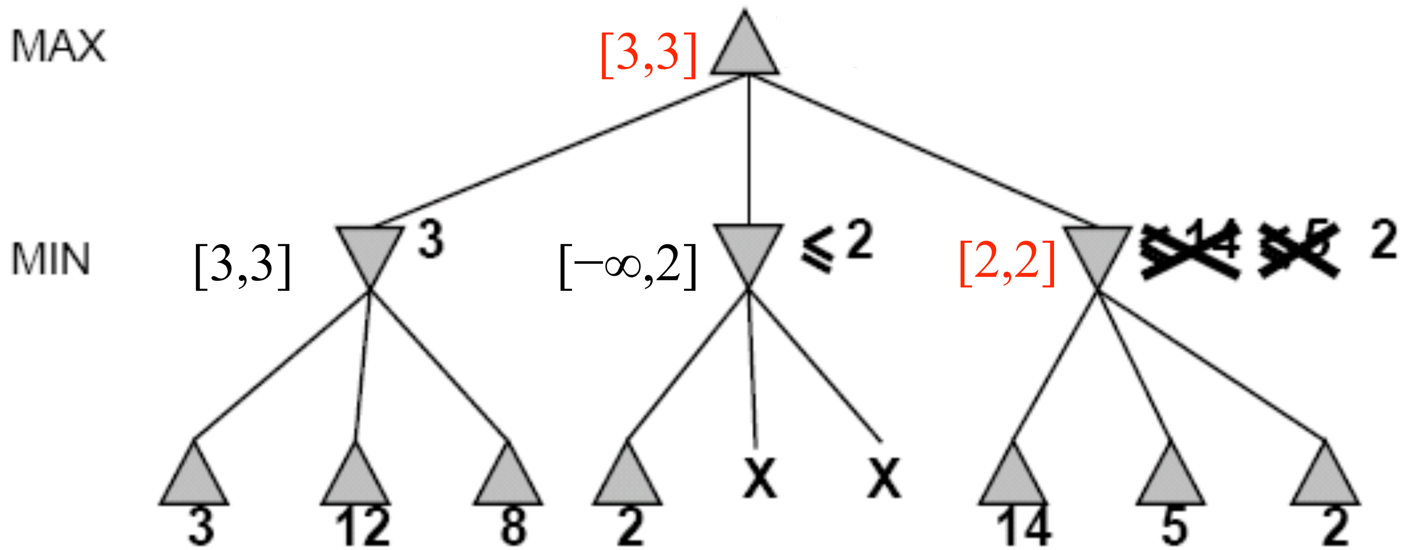
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



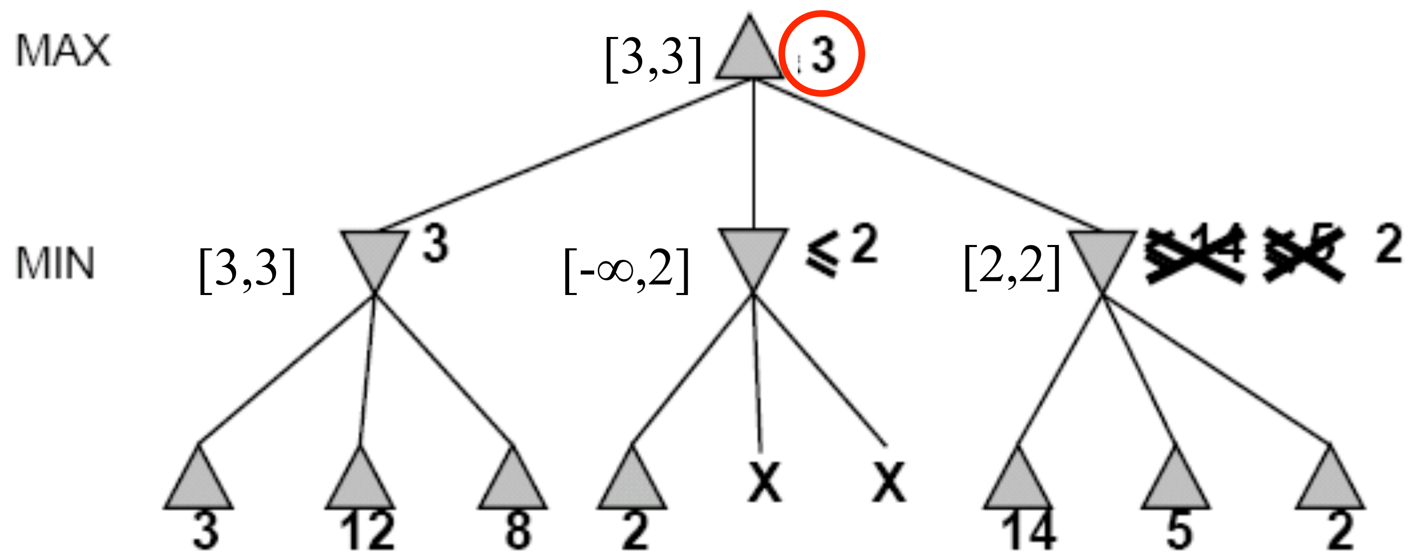
Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



Alpha-Beta Example (continued)

alpha = best choice we've found so far for MAX
beta = best choice we've found so far for MIN



```
def maxValue(state, alpha, beta):
  if state is terminal:
    return utility(state)
  else:
    value = -∞
    for all actions a in actions(state):
      value = max(value, minValue(result(state,a), alpha, beta)
      if value >= beta:
        return value # prune!
      alpha = max(alpha, value) # update alpha
    return value
```

We're making a decision for MAX.

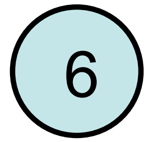
- When considering the MIN's choices, if we find a value that is greater than beta, stop, because MIN won't make this choice
- if we find a better path than alpha, update alpha

```
def minValue(state, alpha, beta):
  if state is terminal:
    return utility(state)
  else:
    value = +∞
    for all actions a in actions(state):
      value = min(value, maxValue(result(state,a), alpha, beta)
      if value <= alpha:
        return value # prune!
      beta = min(beta, value) # update alpha
    return value
```

We're making a decision for MIN.

- When considering the MAX's choices, if we find a value that is less than alpha, stop, because MAX won't make this choice
- if we find a better path than beta for MIN, update beta

Baby NIM2: take 1, 2 or 3 sticks



Effectiveness of pruning

- Notice that as we gain more information about the state of things, we're more likely to prune
- What affects the performance of pruning?
  - key: which order we visit the states
  - can try and order them so as to improve pruning

Effectiveness of pruning

- If perfect state ordering:
  - $O(b^m)$ becomes $O(b^{m/2})$
  - We can solve a tree twice as deep!
- Random order:
  - $O(b^m)$ becomes $O(b^{3m/4})$
  - still pretty good
- For chess using a basic ordering
  - Within a factor of 2 of $O(b^{m/2})$