CS161 - Homework 5
Due: Thursday August 6, 5pm

For all of the problems below **state your running time**. You **do not** need to provide pseudocode, but make sure your algorithm description is clear and understandable. As always, you will be graded on efficiency.

Your proofs should be compelling and convincing, but do not need to be pages long.

Greedy

1. (10 points) - CLRS 16.2-5 (pg. 384) - "Argue" = prove.

Dynamic Programming

2. (15 points) - You are given a string of characters $S = s_1, s_2, ..., s_n$ where all non-alphabetic characters have been removed (e.g. "thisisasentencewithoutanyspacesorpunctuation") and a function $\textsc{dict}(w, i, j)$, which takes as input a string w and two indices i and j and returns $true$ if the string $w_{i...j}$ is a dictionary word and $false$ otherwise.

    (a) (10 points) Give a solution that determines whether the string S consists of a sequence of valid dictionary words. Assume calls to $\textsc{dict}$ are $O(1)$.

    (b) (5 points) If the string is valid, describe how to modify your algorithm to output the sequence of words.

3. (10 points) - Given a sequence of characters $s_1, s_2, ..., s_n$ determine an algorithm that finds the longest subsequence that is the same whether read left to right or right to left. For example, given the sequence

$$A, C, G, T, G, T, C, A, A, A, A, T, C, G$$

the longest such subsequence is A, C, G, T, G, C, A and two non-optimal such subsequences are G, T, T, G and A, T, A. Note that a subsequence is not necessarily contiguous.

Greedy or DP

One of the problems below can be solved more efficiently using a greedy approach and the other cannot (i.e. you must use dynamic programming). For the greedy problem, prove that your solution is optimal.

4. (10 points) You're going on a road trip with friends. Unfortunately, your headlights are broken, so you can only drive in the daytime. Therefore, on any given day you can drive no more than d miles. You have a map with n different hotels and the distances from your start point to each hotel $x_1 < x_2 < ... < x_n$. Your final destination is the last hotel. Describe an algorithm that determines which hotels you should stay in if you want to minimize the number of days it takes you to get to your destination.

5. (10 points) Same setup as above, however, you also want to do sight-seeing along the way. To make sure you don't spend too little or too much time in any one place, you decide to add a penalty for having too much free time. If you travel x miles in a day, then the penalty for that day is $(d - x)^2$. Describe an algorithm that determines the hotel sequence that mimizes the total pentalty, that is the sum of the daily penalties over all travel days.

Extra Credit

6. (5 points) - Given integers $a_1, a_2, ..., a_n$ determine whether it is possible to partition the integers into three disjoint subsets such that the sum of the numbers of each subset is the same. For example, given $(1, 2, 3, 4, 4, 5, 8)$ the answer is *yes* and the partition is $(1, 8)$, $(4, 5)$ and $(2, 3, 4)$ and given $(2, 2, 3, 5)$ the answer is no.

   Express your run time with respect to n and $W = \sum_i a_i$.